74. On Formal Groups over Complete Discrete Valuation Rings. II

Generic Formal Group and Specializations

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1. Let $Z[A_1, A_2, \cdots, A_t, \cdots]$ be the ring of polynomials in countably infinite variables over Z. Let

$$F_A(X, Y)=X+Y+\sum_{i+j\geqq 2} c_{ij}X^iY^j$$

be a commutative formal group over $Z[A_1, A_2, \cdots, A_t, \cdots]$.

Let $a_i \in R(i=1,2,\cdots)$, R being as in [5], and let

$$\varphi: Z[A_1, A_2, \cdots, A_t, \cdots]\longrightarrow R$$

be a ring homomorphism defined by $\varphi(A_i)=a_i$ and $\varphi(d)=d$ if d is in Z. Let

$$\varphi_*F_A(X, Y)=X+Y+\sum_{i+j\geqq 2}\varphi(c_{ij})X^iY^j.$$

Then $\varphi_*F_A(X, Y)$ is a formal group over R. We shall call $\varphi_*F_A(X, Y)$ *a specialization of the generic formal group* $F_A(X, Y)$.

In general, let A, B be commutative rings. Let $\lambda: A\to B$ be a ring homomorphism, $G(X, Y)$ formal power series with coefficients in A. We denote the formal power series obtained from $G(X, Y)$ applying the homomorphism λ to the coefficients of $G(X, Y)$ by $\lambda_*G(X, Y)$ (cf. [1]).

We shall consider $F_A(X, Y)$ and $a_i \in R$, consequently also $\varphi_*F_A(X, Y)$, as fixed, and denote this $\varphi_*F_A(X, Y)$ simply by $F(X, Y)$. If we reduce the coefficients of $F(X, Y)$ mod $\mathfrak{p}$, we obtain a formal group over k which we denote with $\bar{F}(X, Y)$.

On the other hand, let g be a polynomial in $Z[A_1, A_2, \cdots, A_t, \cdots]$. We define $\psi(g)$ to be the polynomial which is obtained from g by reducing its coefficients mod p. Then

$$\psi: Z[A_1, A_2, \cdots, A_t, \cdots]\longrightarrow F_p[A_1, A_2, \cdots, A_t, \cdots]$$

is a ring homomorphism, $\psi_*F_A(X, Y)$ is a commutative formal group over $F_p[A_1, A_2, \cdots, A_t, \cdots]$. Denote this $\psi_*F_A(X, Y)$ by $\bar{F}_A(X, Y)$. If we denote with $\bar{\varphi}$ the ring homomorphism $F_p[A_1, A_2, \cdots, A_t, \cdots]\to k$ defined by $\bar{\varphi}(A_i)=a_i$ mod $\mathfrak{p}$ and $\bar{\varphi}(\bar{d})=\bar{d}$ if $\bar{d}$ is in F_p, we have clearly $\bar{\varphi}_*\bar{F}_A(X, Y)=\bar{F}(X, Y)$.

Let us define $[m]_A(X)=F_A([m-1]_A(X), X)$, $\overline{[m]}_A(X)=\bar{F}_A(\overline{[m-1]}_A(X), X)$, and $\overline{[m]}(X)=\bar{F}(\overline{[m-1]}(X), X)$, inductively like $[m](X)$ in [5].

Then the following diagram (D) is commutative.