73. 2-Dimensional Periodic Continued Fractions and 3-Dimensional Cusp Singularities

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2-dimensional cusp singularities are in one-to-one correspondence with periodic continued fractions, which may be interpreted as cycles of integers. We regard a cycle of integers, as a triangulation of a circle on each vertex of which an integer is attached. Then as a generalization of a periodic continued fraction to dimension 2, we consider a triangulation of a compact topological surface on each edge of which a pair of integers is attached. We show that if it satisfies some conditions, then it induces a 3-dimensional cusp singularity in a manner similar to the 2-dimensional case. Then the singularity has a resolution whose exceptional set is completely determined by the given triangulation realized as the "dual graph". The cusp singularities thus obtained have a duality among themselves generalizing that of Nakamura [2]. In the special case of real tori, we get Hilbert modular cusp singularities.

The details will appear elsewhere.

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Results. Let $N=\boldsymbol{Z}^n$ and $N_{\boldsymbol{R}}=N\otimes_{\boldsymbol{Z}}\boldsymbol{R}\simeq\boldsymbol{R}^n$. Let $\pi: N_{\boldsymbol{R}}\setminus\{0\}\to S^{n-1}$ be the natural projection onto a sphere $S^{n-1}=(N_{\boldsymbol{R}}\setminus\{0\})/\boldsymbol{R}_{>0}$. Then $\mathrm{Aut}(N)=GL(N)$ acts on S^{n-1} through π. Let $\mathcal{S}$ be the set of the pairs (C,Γ) of a cone C in $N_{\boldsymbol{R}}$ and a subgroup Γ of $GL(N)$ satisfying the following conditions: C is open, nondegenerate (i.e., $\overline{C}\cap(\overline{-C})=\{0\}$), convex and Γ-invariant. Moreover, the induced action of Γ on $D=\pi(C)=C/\boldsymbol{R}_{>0}$ is properly discontinuous and fixed point free with the compact quotient D/Γ.

Let $T_N=N\otimes_{\boldsymbol{Z}}\boldsymbol{C}^*\simeq(\boldsymbol{C}^*)^n$ and let $\mathrm{ord}=-\log|\ |: T_N\to N_{\boldsymbol{R}}=T_N/CT_N$ be the canonical map, where CT_N is the compact real torus $N\otimes_{\boldsymbol{Z}}U(1)\simeq U(1)^n$. Using the theory of torus embeddings [2] we can show the following:

Theorem 1. *If (C,Γ) is in $\mathcal{S}$, then we have an n-dimensional cusp singularity* $(V,p)=\mathrm{Cusp}(C,\Gamma)$ *such that* $V\setminus\{p\}\simeq\mathrm{ord}^{-1}(C)/\Gamma$.

Let $\mathcal{T}=\{\mathrm{Cusp}(C,\Gamma)\,|\,(C,\Gamma)\in\mathcal{S}\}$. We have a duality in $\mathcal{T}$ in the following way: Let C^* be the dual cone of C in the dual vector space $M_{\boldsymbol{R}}=N_{\boldsymbol{R}}^*$ of $N_{\boldsymbol{R}}$. Then Γ also acts on M and C^* canonically and (C^*,Γ)