71. *The Corona Problem on 2-Sheeted Disks*

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Consider the normed ring $H^\infty(R)$ of bounded analytic functions on a Riemann surface R equipped with the supremum norm. We denote by $M(R)$ the maximal ideal space of $H^\infty(R)$. We may identify each maximal ideal in $M(R)$ with a multiplicative functional χ on $H^\infty(R)$ with $\chi(1)=1$. An evaluation χ_p at $p\in R$ is a functional on $H^\infty(R)$ given by $\chi_p(f)=f(p)$. The natural map $\tau: R\to M(R)$ is defined by $\tau(p)=\chi_p$. The map τ is injective if and only if $H^\infty(R)$ separates the points in R and in this case we may identify $\tau(R)$ with R. We say that the corona theorem holds for R if $\tau(R)$ is dense in $M(R)$. Carleson [1] proved that the corona theorem holds for the 1-sheeted disk. We will prove the same for 2-sheeted disks. Namely

Theorem. *The corona theorem holds for any unbounded two sheeted covering surface of the unit disk.*

This provides us with nontrivial examples of Riemann surfaces of infinite genus for which the corona theorem holds. The proof will be given below in §§ 1–5.

1. Let (R, D, π) be an unbounded two sheeted covering surface of the unit disk $D=\{|z|<1\}$ with a covering map π so that $\pi^{-1}(z)$ consists of either two different points in R or a single point for each $z\in D$. If R has only a finite number of branch points, then R is conformally equivalent to an interior of a compact bordered Riemann surface for which the validity of the corona theorem is well known (cf. e.g. Gamelin [2]). Therefore we assume that the projection of branch points forms an infinite sequence $\{\alpha_n\}$ $(n=1,2,\cdots)$. Of course the proof given below is also valid for the case where the above sequence is finite.

2. Suppose first the case $\sum(1-|\alpha_n|)=+\infty$. Take an arbitrary $f\in H^\infty(R)$ and consider the $c\in H^\infty(D)$ defined by $c(z)=(f(z_1)-f(z_2))^2$ for $z\in D$ where $\pi^{-1}(z)=\{z_1, z_2\}$. Since $c(\alpha_n)=0$ $(n=1,2,\cdots)$, the Blaschke theorem assures that $c\equiv 0$ and a fortiori $H^\infty(R)=H^\infty(D)\circ\pi$. Therefore $M(R)=M(D)$ and $\tau(R)=D$, and the Carleson theorem [1] assures that $\bar{D}=M(D)$, i.e., $\overline{\tau(R)}=M(R)$.

3. We proceed to the case where $\sum(1-|\alpha_n|)<+\infty$. Let B be the Blaschke product with $\{\alpha_n\}$ its simple zero set. For simplicity we identify $H^\infty(D)\circ\pi$ with $H^\infty(D)$ so that we view $H^\infty(D)\subset H^\infty(R)$. For