70. *Retraction and Extension of Mappings of M_1-Spaces*

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In this paper, we shall prove that an M_1-space X can be imbedded in an M_1-space $Z(X)$ as a closed subset in such a way that X is an AR ($\mathcal{M}_1$) (resp. ANR ($\mathcal{M}_1$)) if and only if X is a retract (resp. neighborhood retract) of $Z(X)$, where $\mathcal{M}_1$ is the class of all M_1-spaces. Moreover, we shall prove that an M_1-space is an AE ($\mathcal{M}_1$) (resp. ANE ($\mathcal{M}_1$)) if and only if it is an AR ($\mathcal{M}_1$) (resp. ANR ($\mathcal{M}_1$)).

Throughout this paper, all spaces are assumed to be Hausdorff topological spaces and all maps to be continuous. N denotes the set of all natural numbers. Let $\mathcal{C}$ be a class of spaces. For the definitions of AR ($\mathcal{C}$), ANR ($\mathcal{C}$), AE ($\mathcal{C}$) and ANE ($\mathcal{C}$), see [4]. Note that in [4] each class $\mathcal{C}$ is weakly hereditary; that is to say, if $\mathcal{C}$ contains X, then it contains every closed subspace of X. However, in this paper we consider the class $\mathcal{M}_1$ of all M_1-spaces though it is unknown if $\mathcal{M}_1$ is weakly hereditary.

1. Auxiliary lemma. For the definitions of uniformly approaching anti-cover and D-space, see [6]. The following lemma was essentially proved in the proof of [5, Lemma, 3.2].

Lemma 1.1. *Let X be a D-space, F a closed subset of X and f a map from F into a space Y. Let Y also denote the natural imbedding of Y in $X\bigcup_f Y=Z$. If $\mathcal{U}=\{U_\alpha : \alpha \in A\}$ is a closure preserving open collection in Y, then for each $\alpha\in A$ there is a collection $\{U'_\beta : \beta\in B_\alpha\}$ of open subsets in Z satisfying the following three conditions:*

(C1) *$\mathcal{U}'=\{U'_\beta : \beta\in B_\alpha,\ \alpha\in A\}$ is closure preserving in Z,*

(C2) *for each $\beta\in B_\alpha$, $U'_\beta\cap Y=U_\alpha$, and for every open subset V in Z with $V\cap Y=U_\alpha$ there is $\beta\in B_\alpha$ such that $U_\alpha\subset U'_\beta\subset V$, and*

(C3) *for every open subset W in Y, there is an open subset W' of Z such that $W'\cap Y=W$ and $W'\cap U'_\beta=\phi$ whenever $\beta\in B_\alpha$ and $W\cap U_\alpha=\phi$.*

Proof. Let p be the projection from the free union $X\cup Y$ to Z. Since X is a D-space, X is an M_1-space. Therefore X is monotonically normal. Let G be a monotone normality operator for X satisfying the properties in [3, Lemma 2.2]. Since X is a D-space, F has a uniformly approaching anti-cover $\mathcal{V}=\{V_\lambda : \lambda\in\Lambda\}$ in X. In particular, since X is hereditarily paracompact, we may assume that $\mathcal{V}$ is locally finite in $X-F$. For each $U_\alpha\in\mathcal{U}$, let $U'_\alpha=\cup\{G(x, F-p^{-1}(U_\alpha)) : x$