69. On Ξ-Product of Spaces which have a σ-Almost Locally Finite Base

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1. **Introduction.** Let $\{X_a : a \in A\}$ be a family of topological spaces. By $B_a X_a$ we denote the set $\prod_a X_a$ with the box product topology. For $p \in B_a X_a$ we denote the subspace $\{x \in B_a X_a : x_a \neq p_a$ for at most finitely many $a\}$ of $B_a X_a$ by Ξ_p.

Recently K. Tamano and the author [3] introduced the notion of almost local finiteness and the class of all spaces which have a σ-almost locally finite base. This class is an intermediate class between that of free L-spaces and that of M_1-spaces. The purpose of this paper is to prove that Ξ_p has a σ-almost locally finite base if each X_a has a σ-almost locally finite base and $p \in B_a X_a$. Corollary 3.2 is an improvement on the result of S. San-ou [5]. By [4], Ξ_p need not be free L even if each X_a is metrizable and $p \in B_a X_a$. For another results on Ξ-product see [1], [2] and [5].

In this paper all spaces are assumed to be regular T_1.

2. **Preliminaries. Definition 2.1.** Let X be a space and $\mathcal{A}$ a family of subsets of X. $\mathcal{A}$ is said to be *almost locally finite* in X if for every point x of X there exist a neighborhood U of x and a finite family $\mathcal{B}$ of subsets of X such that

$\{A \cap U : A \in \mathcal{A}\} \subset \{B \cap W : B \in \mathcal{B}$ and W is a neighborhood of $x\}$.

Lemma 2.2. *Let $\{X_e : e \in E\}$ be a family of spaces and $p \in B_e X_e$. For each $e \in E$ let $\mathcal{A}_e$ be an almost locally finite family of open sets of X_e such that*

$$\text{if } V \in \mathcal{A}_e \text{ then } p_e \in V \text{ or } p_e \notin \mathrm{Cl}\, V.$$

Then $\{\Xi_p \cap \prod_e V_e : (V_e)_{e \in E} \in \prod_e \mathcal{A}_e\}$ is almost locally finite in Ξ_p.

Proof. Let $x \in \Xi_p$.

Case 1. $x = p$.

For each $e \in E$ put $U_e = X_e - \bigcup\{\mathrm{Cl}\, V : V \in \mathcal{A}_e, p_e \notin \mathrm{Cl}\, V\}$. Put $U = \Xi_p \cap \prod_e U_e$. Then U is a neighborhood of x. Let $(V_e)_{e \in E} \in \prod_e \mathcal{A}_e$ and $U \cap \prod_e V_e \neq \emptyset$. Then $x_e = p_e \in V_e$, $e \in E$. Therefore $U \cap \prod_e V_e$ is a neighborhood of x.

Case 2. $x \neq p$.

Let $E_1 = \{e \in E : x_e = p_e\}$ and $E_2 = E - E_1$. Then $|E_2| < \aleph_0$. For $e \in E_1$ put $U_e = X_e - \bigcup\{\mathrm{Cl}\, V : V \in \mathcal{A}_e, p_e \notin \mathrm{Cl}\, V\}$. For $e \in E_2$ there exist