# 68. Nonlinear Perron-Frobenius Problem

## An Extension of Morishima's Theorem

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**1. Introduction.** In connection with the discrete version of Boltzmann equation and Volterra's ecology equation, many researches have been done on the homogeneous quadratic differential equation of the form:

$$\frac{du_j}{dt}=A_j(u_1, \cdots, u_n), \qquad j=1, \cdots, n$$

where each $A_j(u)$ is a quadratic form (cf. Jenks [5], Carleman [6]).

The important first approach to this type of equations is to find their positive equilibrium points, i.e., the equilibrium points whose coordinates are all positive.

The author studied the special case of this equation:

$$\frac{du_j}{dt}=A_j(u_1, \cdots, u_n)-\mu u_j^2, \qquad j=1, \cdots, n$$

where each $A_j(u)$ is a quadratic form with non-negative coefficients. In this case the search of positive equilibrium points is converted to the eigen-value problem of the form:

$$H(u)=\lambda u$$

where $H_j(u)=[A_j(u)]^{1/2}$ and $\lambda=\mu^{1/2}$.

The author was later informed that the theoretical economists had done a great contribution to this type of problems in connection with the balanced growth problem (cf. Morishima [1], Nikaido [2]), and that in the case of Banach spaces, there is a book of Krasnoselskii [4].

But the newly introduced notion of non-sectionality is wider than the indecomposability defined by the economists, and not yet known to the researchers of the Boltzmann equation. So the author hopes that it is useful for both mathematicians and economists.

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**2. Definitions and Notations.** We use vector inequalities. $x\leqq y$ means $x_j\leqq y_j$ (for all $j=1, \cdots, n$), and $x<y$ implies $x_j<y_j$ (for all $j=1, \cdots, n$). And $x\leq y$ implies $x\leqq y$ and $x\neq y$.