60. *On Formal Groups over Complete Discrete Valuation Rings. I*

By Keiichi OSHIKAWA
Department of Mathematics, Musashi Institute of Technology

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1. Introduction. Let R be a complete discrete valuation ring, K the quotient field of R, $\mathfrak{p}$ the maximal ideal of R, π a generator of $\mathfrak{p}$. Put $R/\mathfrak{p}=k$. We assume that the characteristic ch(K) of K is 0, and ch$(k)=p$. We denote with ν the additively written valuation of K with $\nu(\pi)=1$. We put $\nu(p)=e$.

Let $F(X, Y)$ be a commutative formal group over R. Let n be any natural number $\geqq 1$. If $u, v \in \mathfrak{p}^n$, then $F(u, v) \in \mathfrak{p}^n$. We shall write $u \dotplus v$ for $F(u, v)$. Thus $\mathfrak{p}^n$ forms a commutative group with this operation $\dotplus$, which will be denoted with $(\mathfrak{p}^n, \dotplus)$. It is well-known that there exists a formal power series $l_F(X) \in K[[X]]$ of the form

$$l_F(X)=\sum_{n=1}^{\infty} c_n X^n, \quad c_1=1, \quad nc_n \in R$$

such that

$$F(X, Y)=l_F^{-1}(l_F(X)+l_F(Y)). \qquad \text{(Cf. Fröhlich [1].)}$$

It is also known that for sufficiently large n, $(\mathfrak{p}^n, \dotplus)$ is mapped isomorphically onto $\mathfrak{p}^n$ (a commutative group with ordinary addition as operation) by l_F, the inverse map being given by l_F^{-1} (cf. [1]).

In this note, we shall give a "precise" value of α, such that this takes place for $n>\alpha$.

This result implies that, if $(\mathfrak{p}^n, \dotplus)$ has a torsion element u, $\nu(u)$ should be bounded by a value depending on F.

In a subsequent note we shall estimate the above value α under the hypothesis that $F(X, Y)$ is a "specialization of a generic formal group" in the sense which will be explained later.

Our results will be then applied to elliptic curves to improve the classical "Theorem of Lutz".

In the sequel $\boldsymbol{Z}, \boldsymbol{Q}, \boldsymbol{Z}_p, \boldsymbol{F}_p$ will mean as usual the ring of rational integers, the rational number field, the ring of p-adic integers and the finite field with p elements, respectively.

The detailed proofs will appear elsewhere.

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2. The properties of $(\mathfrak{p}, \dotplus)$. For natural number $m \geqq 2$, let us