44. *Curvature and Stability of Vector Bundles**)

By Shoshichi KOBAYASHI**)
Department of Mathematics, University of California, Berkeley

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The purpose of this note is to give a differential geometric condition for a holomorphic vector bundle to be stable or semi-stable.

1. Curvature and the Einstein condition. Let E be a holomorphic vector bundle of rank r over a compact complex manifold M of dimension n and let h be a hermitian structure in E. With respect to a system of linearly independent local holomorphic sections $s_1, \cdots, s_r$ of E, h is given by

(1.1) $$h_{i\bar{j}}=h(s_i, s_j), \quad i, j=1, \cdots, r.$$

Let $z^1, \cdots, z^n$ be a local coordinate system in M. Then the curvature of h is given by

(1.2) $$R_{i\bar{j}\alpha\bar{\beta}}=-\partial_\alpha\partial_{\bar{\beta}}h_{i\bar{j}}+h^{a\bar{b}}\partial_\alpha h_{i\bar{b}}\cdot\partial_{\bar{\beta}}h_{a\bar{j}}, \quad 1\leqq i, j\leqq r, \quad 1\leqq\alpha, \beta\leqq n,$$

where $\partial_\alpha=\partial/\partial z^\alpha$, $\partial_{\bar{\beta}}=\partial/\partial\bar{z}^\beta$, $(h^{a\bar{b}})$ is the inverse matrix of $(h_{i\bar{j}})$, and the summation sign with respect to $a, b=1, \cdots, r$ is omitted. The Ricci tensor of h is given by

(1.3) $$R_{\alpha\bar{\beta}}=h^{i\bar{j}}R_{i\bar{j}\alpha\bar{\beta}}.$$

Then the first Chern class $c_1(E)$ of E is represented by the closed form

(1.4) $$\frac{\sqrt{-1}}{2\pi}R_{\alpha\bar{\beta}}dz^\alpha\wedge d\bar{z}^\beta.$$

Now, in addition to a hermitian structure h in E, we fix a Kähler metric

(1.5) $$g=2g_{\alpha\bar{\beta}}dz^\alpha d\bar{z}^\beta$$

on M. The associated Kähler form is given by

(1.6) $$\Phi=\sqrt{-1}\,g_{\alpha\bar{\beta}}dz^\alpha\wedge d\bar{z}^\beta.$$

The inverse matrix of $(g_{\alpha\bar{\beta}})$ is denoted by $(g^{\alpha\bar{\beta}})$. We set

(1.7) $$K_{i\bar{j}}=g^{\alpha\bar{\beta}}R_{i\bar{j}\alpha\bar{\beta}}.$$

By definition, $h=(h_{i\bar{j}})$ defines a hermitian bilinear form in each fibre of E. Similarly, $K=(K_{i\bar{j}})$ defines also a hermitian bilinear form in each fibre of E. We say that (E, h, M, g) satisfies the *Einstein condition* with factor φ if

(1.8) $$K_{i\bar{j}}=\varphi h_{i\bar{j}},$$

where φ is a real differentiable function on M.

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