42. *The Probabilistic Treatment of Phase Separations in Ising Model with Free Boundary Condition*

By Koji KURODA
Department of Mathematics, Faculty of Science and Technology, Keio University

(Communicated by Kôsaku YOSIDA, M. J. A., April 12, 1982)

Introduction. In this paper we consider the problem of phase separations in Ising model with free boundary condition. As for the pure boundary condition this problem was considered by Minlos and Sinai [1], [2]. Let V be the square in Z^2 with the area $|V|$. Consider the $(+)$-boundary condition and fix the number of minus spins $\rho|V|$ $(0<\rho<1)$. Then they showed that a typical configuration has one connected component of minus spins with the shape of nearly square whose width is about $\rho^{1/2}|V|^{1/2}$.

On the other hand, this fact does not hold in the case of free boundary condition, and the following conjecture was given in [3]; a typical configuration has just one open contour λ which separates V into two parts which are occupied by the opposite phases and λ should be shortest under the condition that V is divided by λ into two regions of volume $\rho|V|$ and $(1-\rho)|V|$. We prove this conjecture positively with respect to the conditional Gibbs measure with free boundary condition.

§1. Ising model with free boundary condition. In this section we give the definition of the conditional Gibbs measure of two-dimensional Ising model with free boundary condition.

Let V be the square in Z^2. Put $\Omega_V=\{+1,-1\}^V$ and $\mathfrak{B}_V=\sigma\{\omega(t);t\in V\}$. For a given configuration $\xi\in\Omega_V$ we draw a unit segment perpendicular to the center of each bond between different spins. Then we have the family of closed lines and open lines. We give these lines the orientations along which we see plus spins on the left side. (See Fig. 1.)

For each line Γ, put $\bar{\Gamma}=(\Gamma,+)$ or $\bar{\Gamma}=(\Gamma,-)$ according that the orientation of Γ is clockwise or anti-clockwise, respectively. We call $\bar{\Gamma}$ open contour if Γ is an open line, and call others closed contours.

It is clear that there is a 1-1 correspondence between the configuration $\xi\in\Omega_V$ and the family of contours $(\bar{\Gamma}_1,\cdots,\bar{\Gamma}_s,\bar{\Delta}_1,\cdots,\bar{\Delta}_k)$, where $\bar{\Gamma}_1,\cdots,\bar{\Gamma}_s$ are closed contours and $\bar{\Delta}_1,\cdots,\bar{\Delta}_k$ are open contours.

The Ising model with free boundary condition is defined by the following probability measure on $(\Omega_V,\mathfrak{B}_V)$,