38. Sharpness of Parametrices for Strictly Hyperbolic Operators

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1. Introduction. Let $P(x,D)$ be a linear partial differential operator with $C^\omega$-coefficients defined in $\mathbb{R}^n$ and strictly hyperbolic with respect to $x$. Let $E_k : \mathcal{D}'(\mathbb{R}) \to \mathcal{D}'(\mathbb{R}^n)$ be $k$-th parametrices, i.e.

$$P(x,D)E_k = 0, \quad D_{z_i}^m E_k|_{z=0} = \delta_{ik} I,$$

where $Y = \{ x \in \mathbb{R}^n ; x_1 = 0 \}$ is the initial plane (see e.g. [1]). We want to study the sharpness of distributions $E_k(x, y) := E_k \delta(x-y)$ here we take $y \in Y$ as parameters. If we take

$$A = A(y) := \{(x, \xi) \in T^* \mathbb{R}^n ; (x, \xi) \text{ is on a bicharacteristic strip through some } (y, \eta) \in T^* \mathbb{R}^n \text{ with } P_\eta(y, \eta) = 0\},$$

$$W = W(y) := \pi A(y),$$

where $\pi : T^* \mathbb{R}^n \to \mathbb{R}^n$ is the natural projection, then we have

$$\text{sing supp } E_k(x, y) \subset W(y).$$

Now take a point $x^0 \in W$ and a component $\omega$ of $\mathbb{R}^n \setminus W$ with $x^0 \in \partial \omega$. Then $E_k(x, y)$ is said to be sharp at $x^0$ from $\omega$ if there is a neighbourhood $V$ of $x^0$ and $u \in C^\omega(V)$ such that $E_k(x, y) = u(x)$ on $\omega \cap V$.

Near each point $x^0 \in W$, $E_k(x, y)$ can be represented by a finite sum of paired oscillatory integrals $I^*(a, \varphi, x)$, for which L. Gårding [3] discovered a criterion for sharpness. But his arguments and proofs are rather sketchily and, in part, incomplete. Our aim is to clarify the situation and to give a rigorous proofs when $x^0 \in W$ is a stable point. Here we use

Definition. $x^0 \in W$ is called a stable point if under small perturbations of $A \subset T^* \mathbb{R}^n$ (as conic Lagrangean manifolds) near $\pi^{-1}(x^0)$, the configurations of $W$ cannot be changed off local diffeomorphisms.

Note that our definition of stability may be considered as a well posedness for the problem of sharpness.

If $\pi^{-1}(x^0) \cap A$ consist of regular points (i.e. $N := \dim T_{x^0} A \cap T_{x^0} (\text{fibre}) = 1$ for $x^0 \in \pi^{-1}(x^0) \cap A$), an easy criterion for sharpness are given in [4]. So, in what follows, we shall consider the case when $\pi^{-1}(x^0) \cap A$ contains irregular points (i.e. the case when $N \geq 2$).

2. Suppose that $\pi^{-1}(x^0) \cap A$ consist of stable and irregular points. Then we can prove that, as a germ at $x^0$, $E_k(x, y)$ can be represented by a finite sum of distributions of the form