35. A Versal Family of Hironaka's Additive Group Schemes

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In connection with the resolution of singularities of algebraic varieties in positive characteristics, Hironaka [1] considered certain subgroup schemes, now called Hironaka's additive group schemes, of the vector group over a field k of characteristic $p>0$. Oda [3] then reduced their study to linear algebra as follows, and together with Mizutani [2] classified them in low dimensions: Hironaka's additive subgroup schemes of exponent not greater than e in an n-dimensional vector group over k are in one-to-one correspondence with the pairs (V, W) consisting of an n-dimensional k^q-vector space W (with $q=p^e$) and a proper k-subspace V of $k\otimes_{k^q}W$ satisfying the condition

(*) $$\mathcal{N}_e\mathcal{D}_e(V)=V$$

(cf. [3, Theorem 2.6]). Moreover, the exponent is exactly e if and only if either $e=0$, or $e>0$ and V is not generated over k by $V\cap(k^p\otimes_{k^q}W)$.

The above condition (*), however, is rather difficult to deal with. We give below alternative formulations of (*), which enable us easily to produce examples and which, hopefully, might turn out to be theoretically useful. Some of our formulations have close connection with the one given by Russel [4].

The details will appear elsewhere.

Results. To deal with all exponents simultaneously and to describe Hironaka's additive group schemes more directly than in [3, § 2], we take the "p^{-e}-th power" of the situation above.

For a nonnegative integer e, let $k_e=k^{1/p^e}$, inside a fixed algebraic closure $\bar{k}$ of k, consist of the p^e-th roots of elements of k and let $k_\infty=\bigcup_{e\geqslant 0}k_e$ be the perfect closure of k. Let $I_\infty\subset k_\infty\otimes_k k_\infty$ (resp. $I_e\subset k_e\otimes_k k_e$) be the kernel of the multiplication map $k_\infty\otimes_k k_\infty\to k_\infty$ (resp. $k_e\otimes_k k_e\to k_e$) so that $I_\infty=\bigcup_{e\geqslant 0}I_e$. Define also the ideal $J_\infty\subset I_\infty$ to be the union

$$J_\infty=\bigcup_{e\geqslant 0}(I_e)^{pe}$$

of the p^e-th power ideals of the ideals I_e. Consider the left k_e-vector space Diff (k_e/k) of differential operators over k of k_e into itself. Then it is naturally a subset of Diff (k_{e+1}/k) via the injection sending $D\in$ Diff (k_e/k) to the operator $k_{e+1}\ni u\mapsto(D(u^p))^{1/p}\in k_{e+1}$. We can thus consider the union

$$\text{Diff}\,(k_\infty/k)=\bigcup_{e\geqslant 0}\text{Diff}\,(k_e/k)$$