34. *A Note on Rings in which Every Maximal Ideal is Generated by a Central Idempotent*

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Introduction. All rings in this note are associative with an identity element and by an ideal we mean a two-sided one. A ring in which every maximal ideal is generated by a central idempotent will be called "A-ring" to simplify the wording.

K. Oshiro [2] has shown by using the representation by global section of a sheaf, that every A-ring satisfying another condition, which is a little complicated to formulate, is Artinian and semisimple and that the converse is also true. In this note we observe that every A-ring is a finite direct sum of simple rings. As the converse is obvious (cf. [1] p. 177), A-rings are nothing but rings of this kind.

The following lemma is evident, and we omit the proof.

Lemma. *If $e\neq 0$ is a central idempotent in a ring R, then eR is a maximal ideal if and only if $(1-e)R$ is a minimal ideal.*

Theorem. *Every A-ring R is a finite direct sum of simple rings.*

Proof. By Lemma, R has a nonzero minimal ideal which is generated by a central idempotent. Now suppose that S is the sum of all minimal ideals each of which is generated by a central idempotent. We claim that $R=S$, for otherwise S is a proper ideal and therefore it must be contained in a maximal ideal M. But we have $M=eR$, where e is a central idempotent and clearly $(1-e)R$ is a minimal ideal. Now $(1-e)R\subseteq S\subseteq M$, a contradiction. Hence $R=S$, and since R has the identity, then R is a finite sum of minimal ideals. Now we claim that R is in fact a finite direct sum of minimal ideals. To see this, put $R=A_1+A_2+\cdots+A_n$, where each A_i is a minimal ideal, then if $A_1+A_2+\cdots+A_n$ is not direct sum, we have $A_i\cap(A_1+A_2+\cdots A_{i-1}+A_{i+1}+\cdots+A_n)\neq(0)$ for some $1\leq i\leq n$ and therefore $A_i\subseteq A_1+A_2+\cdots+A_{i-1}+A_{i+1}+\cdots+A_n$. If we repeat this process a finite number of times, we get $R=R_1\oplus R_2\oplus\cdots\oplus R_k$, where each R_i is a minimal ideal of R and it is easy to see that each R_i is a simple ring, for each ideal of R_i is also an ideal of R.