33. On Regular Elliptic Conjugacy Classes of the Siegel Modular Group

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1. Introduction. In this paper we announce two theorems on regular elliptic conjugacy classes of the Siegel modular group of degree $2n$. The detailed discussion with proof will appear elsewhere. For the general modular group $GL(n, \boldsymbol{Z})$, it was shown by C. G. Latimer and C. C. MacDuffee [3] and O. Taussky [4] that the number of conjugacy classes, which have an irreducible characteristic polynomial, is equal to the number of ideal classes of a subring in a certain algebraic number field. Especially, if the characteristic polynomial of a conjugacy class is a cyclotomic polynomial f, then that ring consists of all algebraic integers in the splitting field of f over $\boldsymbol{Q}$.

Let $\Gamma = Sp(2n, \boldsymbol{Z})$ be the Siegel modular group of degree $2n$. Concerning the conjugacy classes of Γ, we get some results analogous to the above mentioned result for $GL(n, \boldsymbol{Z})$. Our results in this paper are an existence proof of the "regular elliptic elements" in Γ and a formula in class number for the "regular elliptic elements" of Γ. We shall state our results more precisely after the preparations in § 2.

2. Preliminaries. Let $G = Sp(2n, \boldsymbol{R})$ be the real symplectic group of degree $2n$. The group G is defined by

$$G = \{g \in GL(2n, \boldsymbol{R})\,;\, {}^t gJg = J\} \tag{2.1}$$

where $J = \begin{pmatrix} 0 & 1_n \\ -1_n & 0 \end{pmatrix}$ and 1_n is the identity matrix of degree n. Let $\mathfrak{S}$ be the set of all positive definite symmetric matrices in G. Then $\mathfrak{S}$ is identified with the Siegel upper half space. The group G acts on $\mathfrak{S}$ by the rule $G \times \mathfrak{S} \ni (g, p) \rightarrow {}^t gpg \in \mathfrak{S}$.

Definition 1. An element g in G is called *elliptic* if g fixes an element in $\mathfrak{S}$.

Let $O(2n)$ be the orthogonal group of degree $2n$ and put $K = O(2n) \cap G$. Then K is a maximal compact subgroup of G. It is easily seen that an element h in G is elliptic if and only if h is conjugate to an element in K.

Let us define a regular element in G. We denote the Lie algebra of G by $\mathfrak{g}$. The adjoint action of G on $\mathfrak{g}$ is defined by

$$Ad(g)X = gXg^{-1}, \quad g \in G, \quad X \in \mathfrak{g}.$$

The rank $r(G)$ of G is defined by the following formula: