31. Classification of Projective Varieties of Δ-Genus One

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Introduction. Let V be a subvariety (=irreducible reduced closed subscheme) of a projective space $\boldsymbol{P}^N$ defined over an algebraically closed field $\mathfrak{K}$ of any characteristic. Set $n=\dim V$, $d=\deg V$ and $m=\operatorname{codim} V=N-n$. In this note we always assume that the restriction mapping $H^0(\boldsymbol{P}^N, \mathcal{O}(1))\to H^0(V, L)$ is bijective, where $L=\mathcal{O}_V(1)$. Then $\Delta=d-m-1=n+d-h^0(V, L)$ is the Δ-genus of the polarized variety (V, L) (cf. [1] etc.).

It is well-known that $\Delta\geqq 0$ for every V as above. Moreover, we have the following

Theorem 0 (see, e.g., [1] if $\operatorname{char}(\mathfrak{K})=0$ and [4] in general). *If $\Delta=0$, then V is one of the following types:*

1) $(\boldsymbol{P}^n, \mathcal{O}(1))$.

2) *A hyperquadric.*

3) *A rational scroll. This means that $(V, L)\cong(\boldsymbol{P}(E), \mathcal{O}(1))$ for an ample vector bundle E on $\boldsymbol{P}^1$.*

4) *A Veronese surface $(\boldsymbol{P}^2, \mathcal{O}(2))$ in $\boldsymbol{P}^5$.*

5) *A generalized cone* (this means that the set of the vertices may be a linear space of positive dimension) *over a projective manifold of one of the above types* 2)–4).

In this note we consider the case $\Delta=1$. Details and proofs will be published elsewhere.

As for non-singular varieties, we have the following

Theorem I (cf. [2] [3] and [4]). *Let V be a projective non-singular variety as above with $\Delta=1$. Then the dualizing sheaf ω_V is isomorphic to $\mathcal{O}_V(1-n)$. Moreover, if $n\geqq 3$, then V is one of the following types:*

1) *A hypercubic.* $d=3$.

2) *A complete intersection of two hyperquadrics.* $d=4$.

3) *A linear section of the Grassmann variety parametrizing lines in $\boldsymbol{P}^4$, embedded by the Plücker coordinate.* $d=5$ *and* $n\leqq 6$.

4) (A hyperplane section of) *the Segre variety $\boldsymbol{P}^2\times\boldsymbol{P}^2$ in $\boldsymbol{P}^8$.* $d=6$.

5) *The Segre variety $\boldsymbol{P}^1\times\boldsymbol{P}^1\times\boldsymbol{P}^1$ in $\boldsymbol{P}^7$.* $d=6$.

6) *The blowing-up of $\boldsymbol{P}^3$ at a point.* $d=7$.

7) *Veronese threefold $(\boldsymbol{P}^3, \mathcal{O}(2))$ in $\boldsymbol{P}^9$.* $d=8$.