30. *Singular Integrals on a Locally Compact Abelian Group with an Action of a Compact Group*

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(Communicated by Kôsaku YOSIDA, M. J. A., March 12, 1982)

Let G be a semidirect product group of a closed normal abelian subgroup A and a compact subgroup K. An action of K on A is given by $k(a)=kak^{-1}$ for all $k\in K$ and $a\in A$. Let $\hat{A}$ be the dual group of A. If $k\in K$ and $\gamma\in\hat{A}$, the equation that $\langle k(\gamma), a\rangle=\langle\gamma, k^{-1}(a)\rangle$ for all $a\in A$, defines an action of K on $\hat{A}$. If E is a finite dimensional vector space, then for $1\leqq p\leqq\infty$, $L^p(A;E)$ will denote the Banach space of all L^p functions on A with values in E. Suppose (λ, E) is a finite dimensional unitary representation of K. A G-action $\tau(g)$ on $L^p(A;E)$ will be defined by $\tau(g)f(a')=\tau(ak)f(a')=\lambda(k)f(k^{-1}(a^{-1}a'))$ for all $a'\in A$ and all $g\in G$ with $g=ak$, $a\in A$, $k\in K$. The *Fourier transform* of f in $L^1(A;E)$ is defined by $\mathfrak{F}f(\gamma)=\int_A\langle\gamma,\overline{a}\rangle f(a)da$ for all $\gamma\in\hat{A}$. We give a definition of a polar decomposition (Σ, C) of A (cf. [3]). Let K_0 be a closed subgroup of K, and let C be a Borel subset of A whose elements are invariant under the action of K_0. Let Σ be the homogeneous space K/K_0. We say that (Σ, C) is a *polar decomposition* of A provided that

(a) for each r in C the stability group of r in K is precisely K_0, and

(b) the mapping $(kK_0, r)\mapsto k(r)$ is a homeomorphism of $\Sigma\times C$ onto a Borel subset A_0 of A whose complement in A is of Haar measure zero in A. To avoid a trivial case we assume that the identity element e of A does not belong to A_0 throughout this paper.

Let $\hat{K}$ be the set of all equivalence classes of irreducible unitary representations of K. For π in $\hat{K}$ we denote by $d(\pi)$ the dimension of π and by $m(\pi)$ the multiplicity with which π occurs in $L^2(\Sigma)$. Le $\tilde{K}$ be the subset of $\hat{K}$ consisting of all elements with $m(\pi)\neq 0$. We set $\tilde{K}_0=\tilde{K}-\{\text{the trivial representation}\}$. Let (π, H_π) be an element of $\tilde{K}$ and let $\{v_j^\pi\}_{j=1,\dots,d(\pi)}$ be a fixed orthonormal basis of H_π such that $\pi(k_0)v_j^\pi=v_j^\pi$, $j=1,\cdots,m(\pi)$ for all k_0 in K_0. We put $Y_{ij}^\pi(k)=\sqrt{d(\pi)}(v_i^\pi, \pi(k)v_j^\pi)$, $i=1,\cdots,d(\pi)$, $j=1,\cdots,m(\pi)$. Then the set of functions $\{Y_{ij}^\pi:\pi\in\tilde{K}, i=1,\cdots,d(\pi), j=1,\cdots,m(\pi)\}$ is an orthonormal basis of $L^2(\Sigma)$. We call the functions Y_{ji}^π *generalized spherical harmonics* in $L^2(\Sigma)$. Throughout this paper we will use a fixed set of generalized spherical harmonics Y_{ij}^π and we assume that A and $\hat{A}$ have polar decompositions