28. *The Nonstationary Navier-Stokes System with Some First Order Boundary Condition*

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Introduction. This paper shows that there exists a strong solution in L_p of the nonstationary Navier-Stokes system with some first order boundary condition. To prove this we study the Stokes operator with such boundary condition and use the semigroup approach in Fujita-Kato [2], [8] and Giga-Miyakawa [7].

Let D be a bounded domain in $\boldsymbol{R}^n$ with smooth boundary S. We consider the Navier-Stokes initial value problem concerning velocity $u=(u^1, \cdots, u^n)$ and pressure p:

(N) $\partial u/\partial t-\Delta u+(u, \nabla)u+\nabla p=0$, $\operatorname{div} u=0$ in $D\times(0, T)$, $u|_{t=0}=a$ in D,

where $(u, \nabla)=\sum_{j=1}^n u^j(\partial/\partial x_j)$. The boundary condition we give is

(NB) $$u\cdot\nu=0, \quad Bu=0 \quad \text{on} \quad S\times(0, T).$$

Here ν_x denotes the interior unit normal vector at $x\in S$ and $u\cdot\nu=u^1\nu^1+\cdots+u^n\nu^n$. We assume that B is a first order boundary differential operator and that $Bu\cdot\nu=0$ if $u\cdot\nu=0$.

To study this Navier-Stokes system in L_p we define the Stokes operator as follows. Let X_p $(1<p<\infty)$ denote the set of divergence free vector functions $w\in L_p(D)$ satisfying $w\cdot\nu=0$. Let P be the continuous projection from $L_p(D)$ to X_p; see [3]. Then we set $A_B=-P\Delta$ with domain $D(A_B)=\{u\in W_p^2(D);\ Bu=0\}\cap X_p$ and call A_B the *Stokes operator* with boundary condition B; here $W_p^2(D)$ denotes the Sobolev space of order two.

Concerning A_B we shall show that $-A_B$ generates an analytic semigroup in X_p if B satisfies an appropriate algebraic assumption (see the assumption (B) in § 1); the slip boundary condition is included in our case. Next we shall characterize $D((A_B+L)^\alpha)$ $(0<\alpha<1)$ for large L. We shall also study A_B^*, the dual of A_B.

Following Kato-Fujita [2], [8], we transform (N), (NB) into the evolution equation in X_p

(AN) $$du/dt+A_Bu+P(u, \nabla)u=0 \quad (t>0), \quad u(0)=a.$$

Using results on A_B, we get the existence and the uniqueness of a (local) strong solution of (AN).

Since our methods are similar to those of Giga [4]–[6] and Giga-Miyakawa [7] who studied the Dirichlet problem for (N), we do not give the detailed proof here. However, our results generalize that of