

14. A Summation Method for Linearly Ordered Series

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By a *linearly ordered sequence* with real-valued terms we understand an arbitrary real-valued function f defined on an indexing set Ω , where we suppose Ω to be nonvoid, at most countable, and furnished with a linear order. We are concerned with a summation method by which, under certain conditions, the terms $f(\omega)$ of a linearly ordered sequence can be summed in the given order of the indices ω . When we deal with summation, we shall speak of a linearly ordered *series* instead of *sequence*, in accordance with the usual wording.

Given Ω as above, there always exists a linear compact set Q of measure zero, containing at least two points and such that Ω can be mapped biuniquely and order-isomorphically onto the aggregate \mathfrak{M} of the open intervals contiguous to Q , where we suppose \mathfrak{M} to be linearly ordered by its natural ordering. This set Q and the relevant order-isomorphism will be kept fixed in the sequel.

We shall say that a linearly ordered series $\sum_{\omega} f(\omega)$ defined on Ω is *Luzin convergent*, if there exists at least one real-valued function $F(x)$ ($-\infty < x < +\infty$) subject to the following three conditions and if, further, two such functions F necessarily differ, over the set Q , only by an additive constant.

- (i) The function $F(x)$ is continuous on the set Q ;
- (ii) the image $F[Q]$ of Q under F is of measure zero;
- (iii) we have $f(\omega) = F(v) - F(u)$, for every $\omega \in \Omega$, where (u, v) is that open interval (contiguous to Q) which corresponds to the index ω under the order-isomorphism considered above.

When this is the case, we define the *Luzin sum* of our series by the formula $\sum_{\omega} f(\omega) = F(b) - F(a)$, where $[a, b]$ denotes the minimal closed interval containing the set Q . Clearly, this definition is independent of the choice of the function F . The epithet "Luzin" indicates the importance of the condition (N) of Luzin in our theory.

It can be shown that the above notion of Luzin convergence, as well as the value of the Luzin sum, depends neither on the choice of the set Q nor on that of the relevant order-isomorphism.

We shall call a linearly ordered series $\sum_{\omega} f(\omega)$ defined on an indexing set Ω to be *Denjoy convergent*, if there exists a real-valued function $F(x)$ ($-\infty < x < +\infty$) which is generalized absolutely continu-