111. On Regularity Properties for some Nonlinear Parabolic Equations*)

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The contents of this paper consist of some amelioration and supplement to the previous paper [4].

Let Ω be a not necessarily bounded domain in R^N, $N>2$, which is uniformly regular of class C^2 and locally regular of class C^4 in the sense of F. E. Browder [1]. The boundary of Ω is denoted by Γ. Let

$$a(u,v)=\int_\Omega\left(\sum_{i,j=1}^N a_{ij}\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_j}+\sum_{i=1}^N b_i\frac{\partial u}{\partial x_i}v+cuv\right)dx$$

be a bilinear form defined in $H^1(\Omega)\times H^1(\Omega)$. The coefficients a_{ij}, b_i are bounded and continuous in $\bar{\Omega}$ together with first derivatives and c is bounded and measurable in Ω. The matrix $\{a_{ij}(x)\}$ is uniformly positive definite in Ω. It is assumed that $c\geqq 0$, $c-\sum_{i=1}^N \partial b_i/\partial x_i\geqq 0$ a.e. in Ω.

Let $j(x,r)$ be a function defined on $\Gamma\times R$ such that for each fixed $x\in\Gamma$ $j(x,r)$ is a proper convex lower semicontinuous function of r and $j(x,r)\geqq j(x,0)=0$. The subdifferential of j with respect to r is denoted by β. We assume that for each $t\in R$ and $\lambda>0$ $(1+\lambda\beta(x,\cdot))^{-1}(t)$ is a measurable function of x (cf. B. D. Calvert-C. P. Gupta [2]). For a function u defined on Γ $j(u)$ denotes the function $j(x,u(x))$, $x\in\Gamma$.

Set

$$\Gamma_1=\{x\in\Gamma:\beta(x,0)=R\},\qquad \Gamma_2=\Gamma\backslash\Gamma_1.$$

Γ_1 is the part of Γ where the boundary condition is of Dirichlet type. We assume that $\sum_{i=1}^N b_i\nu_i\geqq 0$ on Γ_2 where $\nu=(\nu_1,\cdots,\nu_N)$ is the outernormal vector to Γ. Set

$$V=\{u\in H^1(\Omega):u=0 \text{ on } \Gamma_1\}.$$

Let $\Psi(x)$ be a function belonging to $H^1(\Omega)\cap L^1(\Omega)$ such that $\Psi\leqq 0$ on Γ_1. We assume that

$$\{u\in V:u\geqq\Psi \text{ a.e.}, j(u|_\Gamma)\in L^1(\Gamma)\}$$

is not empty, or equivalently $j(\Psi^+|_\Gamma)\in L^1(\Gamma)$.

The norm of $L^2(\Omega)$ and $H^1(\Omega)$ are denoted by $|\ \ |$ and $\|\ \ \|$ respectively. The inner product of $L^2(\Omega)$ as well as the pairing between V and V^* are both denoted by $(\ ,\ )$. The norm of $L^p(\Omega)$ is denoted by $|\ \ |_p$.

The mapping A which is multivalued in general is defined as fol-

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