80. The τ Function of the Kadomtsev-Petviashvili Equation

Transformation Groups for Soliton Equations. I

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The notion of τ function was first introduced by Sato, Miwa and Jimbo in a series of papers on holonomic quantum fields [1]. There the τ functions were simply expressed as the expectation values of field operators which belong to the Clifford group of free fermions. Then by several authors [2]-[4] the concept was generalized and exploited in the study of monodromy and spectrum preserving deformations. They also showed that τ functions are nothing other than the dependent variables used by Hirota in this theory of bilinear equations [5].

This is the first in a series of papers [6], [7] by the present authors, E. Date and M. Jimbo, which aims at a further study of τ functions and soliton equations.

The main results in the present paper are the following. a) We construct a Clifford operator $\varphi(x)$ so that for any even Clifford group element g the expectation value $\tau(x)=\langle\varphi(x)g\rangle$ gives us a solution to the hierarchy of the KP (Kadomtsev-Petviashvili) equations in Hirota's bilinear form. b) Define polynomials $p_j(x)$ $(j=0,1,2,\cdots)$ by

$$\exp\left(\sum_{j=1}^{\infty} k^j x_j\right)=\sum_{j=0}^{\infty} p_j(x)k^j.$$

The KP hierarchy contains the following infinite number of bilinear equations:

$$\det\begin{bmatrix} p_{f_1+1}\left(-\frac{\tilde{D}}{2}\right) & p_{f_1+1}\left(\frac{\tilde{D}}{2}\right) & \cdots p_{f_1+m-1}\left(\frac{\tilde{D}}{2}\right) \\ p_{f_2}\left(-\frac{\tilde{D}}{2}\right) & p_{f_2}\left(\frac{\tilde{D}}{2}\right) & \cdots p_{f_2+m-2}\left(\frac{\tilde{D}}{2}\right) \\ \vdots & \vdots & \vdots \\ p_{f_m-m+2}\left(-\frac{\tilde{D}}{2}\right) & p_{f_m-m+2}\left(\frac{\tilde{D}}{2}\right) & \cdots p_{f_m}\left(\frac{\tilde{D}}{2}\right) \end{bmatrix}\tau(x)\cdot\tau(x)=0,$$

$(f_1\geq f_2\geq\cdots\geq f_m\geq 1,\ m\geq 2,\ \tilde{D}=(D_1,\ D_2/2,\ D_3/3,\cdots))$.

Our work is deeply related to the recent progress [8] by M. Sato and Y. Sato on the structure of τ functions for the KP hierarchy. In fact, our starting point was Sato's lecture [9] in which he claimed