80. The $\tau$ Function of the Kadomtsev-Petviashvili Equation

Transformation Groups for Soliton Equations. I

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The notion of $\tau$ function was first introduced by Sato, Miwa and Jimbo in a series of papers on holonomic quantum fields [1]. There the $\tau$ functions were simply expressed as the expectation values of field operators which belong to the Clifford group of free fermions. Then by several authors [2]-[4] the concept was generalized and exploited in the study of monodromy and spectrum preserving deformations. They also showed that $\tau$ functions are nothing other than the dependent variables used by Hirota in this theory of bilinear equations [5].

This is the first in a series of papers [6], [7] by the present authors, E. Date and M. Jimbo, which aims at a further study of $\tau$ functions and soliton equations.

The main results in the present paper are the following. a) We construct a Clifford operator $\varphi(x)$ so that for any even Clifford group element $g$ the expectation value $\tau(x) = \langle \varphi(x)g \rangle$ gives us a solution to the hierarchy of the KP (Kadomtsev-Petviashvili) equations in Hirota's bilinear form. b) Define polynomials $p_i(x) (i = 0, 1, 2, \ldots)$ by

$$\exp \left( \sum_{j=1}^{\infty} k^j x_j \right) = \sum_{j=0}^{\infty} p_j(x)k^j.$$

The KP hierarchy contains the following infinite number of bilinear equations:

$$\det \begin{pmatrix} p_{f_1+1}(-\tilde{D}/2) & p_{f_1+1}(\tilde{D}/2) & \cdots & p_{f_1+m-1}(-\tilde{D}/2) \\ p_{f_2}(-\tilde{D}/2) & p_{f_2}(\tilde{D}/2) & \cdots & p_{f_2+m-2}(-\tilde{D}/2) \\ \vdots & \vdots & \ddots & \vdots \\ p_{f_{m-m+2}}(-\tilde{D}/2) & p_{f_{m-m+2}}(\tilde{D}/2) & \cdots & p_{f_{m}}(\tilde{D}/2) \end{pmatrix} \cdot \tau(x) \cdot \tau(x) = 0,$$

$(f_1 \geq f_2 \geq \cdots \geq f_m \geq 1, m \geq 2, \tilde{D} = (D_1, D_2/2, D_3/3, \cdots))$.

Our work is deeply related to the recent progress [8] by M. Sato and Y. Sato on the structure of $\tau$ functions for the KP hierarchy. In fact, our starting point was Sato’s lecture [9] in which he claimed...