70. *An Analogue of Paley-Wiener Theorem on a Real Rank 2 Semisimple Lie Group*

The Case of 1 Dimensional τ-Sherical Functions

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In the previous paper [4] we obtained an analogue of Paley-Wiener theorem on $SU(2,2)$. In this article we shall give more precise results about this theorem, particularly, replace the condition (C1) in [4] by explicit conditions (cf. (C2)–(C4) in § 5).

1. Notation and assumptions. Let G be a connected semisimple Lie group with finite center and $G=KAN$ an Iwasawa decomposition for G. Let M be the centralizer of A in K and put $P=MAN$. Then P is a minimal parabolic subgroup of G. We denote the Lie algebras by small German letters. Let Σ denote the set of all roots for the pair $(\mathfrak{g}, \mathfrak{a})$ and W the corresponding Weyl group. Let Σ^+ denote the set of all positive roots in Σ and $\mathfrak{a}^+$ the corresponding positive Weyl chamber in $\mathfrak{a}$. Put $\rho=(1/2)\sum_{\beta\in\Sigma^+}\beta$. For simplicity we denote the dual space of $\mathfrak{a}$ by $\mathcal{F}$ and its complexification by $\mathcal{F}^c$. Put $\mathcal{F}^+=\{\lambda\in\mathcal{F};\ \langle\lambda,\alpha\rangle>0$ for all $\alpha\in\Sigma^+\}$ and $A^+=\exp\mathfrak{a}^+$.

For any root α in Σ let $\mathfrak{a}_\alpha$ denote the hyperplane of $\alpha=0$ in $\mathfrak{a}$ and put $A_\alpha=\exp\mathfrak{a}_\alpha$. Let L_α denote the centralizer of A_α in G. Then it is easy to see that $L_\alpha=M_\alpha A_\alpha$, where $M_\alpha=\bigcap_{\chi\in X(L_\alpha)}\operatorname{Ker}|\chi|$ ($X(L_\alpha)$ is the group of all continuous homomorphisms of L_α into the multiplicative group of real numbers). Then we can define the parabolic subgroup $P_\alpha=M_\alpha A_\alpha N_\alpha$ such that $N_\alpha\subset N$. Put ${}^*P_\alpha=P\cap M_\alpha$ and ${}^*A_\alpha=A\cap M_\alpha$, ${}^*N_\alpha=N\cap M_\alpha$. Then it is easy to see that ${}^*P_\alpha=M{}^*A_\alpha{}^*N_\alpha$ is a minimal parabolic subgroup of M_α and $\dim{}^*A_\alpha=1$. For this pair $(M_\alpha, {}^*A_\alpha)$ we define ${}^*\rho$, ${}^*\mathcal{F}_\alpha$, ${}^*\mathcal{F}_\alpha^c$, ${}^*\mathcal{F}_\alpha^+$ and ${}^*A_\alpha^+$ by the same way, where $\lambda({}^*a)>0$ for ${}^*a\in{}^*A_\alpha^+$ and $\lambda\in\mathcal{F}^+$.

Let $\tau=(\tau_1,\tau_2)$ be a unitary double representation of K on a finite dimensional Hilbert space V. Let $\mathcal{C}(G,\tau)$ denote the τ-spherical Schwartz space on G and ${}^\circ\mathcal{C}(G,\tau)$ the closed subspace of $\mathcal{C}(G,\tau)$ consisting of all cusp forms. For any parabolic subgroup $Q=M_QA_QN_Q$ of G let $\mathcal{C}_{A_Q}(G,\tau)$ denote the closed subspace of $\mathcal{C}(G,\tau)$ consisting of all wave packets corresponding to Q (cf. [2, § 26]). Let τ_M and $\tau_{M_\alpha}(\alpha\in\Sigma^+)$ denote the restrictions of τ to M and $M_\alpha\cap K$ respectively. Then for