57. *Singular Hadamard's Variation of Domains and Eigenvalues of the Laplacian. II*

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§ 1. This paper is a continuation of our previous note [2]. Let Ω be a bounded domain in $\mathbf{R}^n$ with C^3 boundary γ and w be a fixed point in Ω. For any sufficiently small $\varepsilon>0$, let B_ε be the ball defined by $B_\varepsilon=\{z\in\Omega\,;\,|z-w|<\varepsilon\}$. Let Ω_ε be the bounded domain defined by $\Omega_\varepsilon=\Omega\setminus\bar{B}_\varepsilon$. Then the boundary of Ω_ε consists of γ and ∂B_ε. Let $0>\mu_1(\varepsilon)>\mu_2(\varepsilon)>\cdots$ be the eigenvalues of the Laplacian with the Dirichlet condition on $\partial\Omega_\varepsilon$. We arrange them repeatedly according to their multiplicities. In [2], [3] we gave the asymptotic formulas for the j-th eigenvalue $\mu_j(\varepsilon)$ when $\varepsilon\searrow 0$ in case $n=2$, 3. In this note we treat the case $n=4$. We have the following

Theorem 1. *Assume $n=4$. Fix j. Suppose that the j-th eigenvalue μ_j of the Laplacian with the Dirichlet condition on γ is a simple eigenvalue, then*

$$\mu_j(\varepsilon)-\mu_j=-2S_4\varepsilon^2\varphi_j(w)^2+O(\varepsilon^{5/2}) \tag{1.1}$$

holds when ε tends to zero. Here φ_j denotes the eigenfunction of the Laplacian with the Dirichlet condition on γ satisfying

$$\int_\Omega \varphi_j(x)^2dx=1.$$

Here S_4 denotes the area of the unit sphere in $\mathbf{R}^4$.

Our aim of this note is to offer a rough sketch of the proof of the above theorem. Calculation and technique which are used to prove Theorem 1 are more elaborate than in case $n=2$ and 3. $L^p(1<p<\infty)$ spaces are used in this note. We employed only L^2 spaces in case $n=2$, 3.

We review a generalization of the Schiffer-Spencer formula. See [6]. Also see [3]. In the following we assume $n=4$. Let $G(x,y)$ be the Green's function on Ω. Put

$$\omega_\varepsilon=\{x\in\Omega\,;\,G(x,w)<(2S_4\varepsilon^2)^{-1}\}$$

and $\beta_\varepsilon=\Omega\setminus\bar{\omega}_\varepsilon$. Let $G_\varepsilon(x,y)$ be the Green's function in ω_ε.

Variational formula for the Green's function [3]. Fix x, $y\in\Omega\setminus\{w\}$ such that $x\neq y$. Then

$$G_\varepsilon(x,y)=G(x,y)-2S_4\varepsilon^2G(x,w)G(y,w)+O(\varepsilon^3) \tag{1.2}$$

holds when ε tends to zero. The remainder term is not uniform with respect to x, y.

To prove Theorem 1 we use the iterated kernel $G_\varepsilon^{(2)}$ (resp. $G^{(2)}$) of