# 44. Another Construction of Lie Algebras by Generalized Jordan Triple Systems of Second Order

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**Introduction.** U. Hirzebruch [3] and G. Rhinow [9] have generalized Tits' construction of Lie algebras by Jordan algebras [11] to Jordan triple systems (JTS), using a certain two dimensional JTS. Moreover H. Asano and K. Yamaguti [2] have generalized Hirzebruch's construction to generalized JTS of second order (due to I. L. Kantor [4]), using the same two dimensional JTS. In this note, it is shown that Lie algebras can be also constructed by generalized JTS of second order (gen. JTS of 2nd order), using a certain two dimensional associative triple system (ATS) (cf. [6]). From a two dimensional triple system $W$ and any gen. JTS $\mathfrak{J}$ of 2nd order, we make a gen. JTS $W\otimes\mathfrak{J}$ of 2nd order, where $W$ is a certain ATS (see § 1) while in [2], $W$ was a certain JTS. In both cases, Lie algebras can be constructed from $W\otimes\mathfrak{J}$. In other words, Lie algebras can be constructed from gen. JTS $(\mathfrak{J}\oplus\mathfrak{J})_\varepsilon$ of 2nd order (see § 2) where in case $\varepsilon=-1$ we have the Asano–Yamaguti construction and in case $\varepsilon=+1$, we obtain our construction in this note. We assume that any vector space considered in this note is finite dimensional and the characteristic of base field $\Phi$ is different from 2 or 3. The author wishes to express his hearty thanks to Prof. K. Yamaguti for his kind advices and encouragements.

**§ 1.** A triple system satisfying $\{ab\{cde\}\}=\{a\{bcd\}e\}=\{\{abc\}de\}=\{a\{dcb\}e\}$ for any elements $a, b, c, d, e$ is called an ATS.

Let $W$ be a two dimensional triple system which has a basis $\{e_1, e_2\}$ such that

(1) $$\{e_1e_1e_1\}=\alpha e_1,\quad \{e_1e_1e_2\}=\{e_1e_2e_1\}=\{e_2e_1e_1\}=\alpha e_2,$$
$$\{e_1e_2e_2\}=\{e_2e_1e_2\}=\{e_2e_2e_1\}=\beta e_1,\quad \{e_2e_2e_2\}=\beta e_2,$$

where $\alpha, \beta\in\Phi$. Then $W$ is a commutative ATS and is also a JTS. In the ATS $W$, we have

(2) $$l(a, b)l(c, d)=l(c, d)l(a, b),$$

(3) $$l(a, b)l(c, d)=l(l(a, b)c, d)=l(c, l(b, a)d),$$

where $l(a, b)c=\{abc\}$, for $a, b, c, d\in W$.

A gen. JTS $\mathfrak{J}$ of 2nd order is a vector space with a triple product $\{xyz\}$ satisfying