36. *Branching of Singularities for Degenerate Hyperbolic Operators and Stokes Phenomena. II*

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Recently, one of the authors revealed a closed connection between branching of singularities and Stokes phenomena for a certain class of degenerate hyperbolic operators ([2]). We generalize his result to the following type of degenerate hyperbolic linear partial differential operators P in $\boldsymbol{R}_t \times \boldsymbol{R}_x^n$:

$$P=\sum_{i=0}^{m} P_{m-i}(t, x, D_t, D_x),$$

$$P_m(t, x, \tau, \xi)=\prod_{i=1}^{m} (\tau-t^{\ell}\lambda_i(t, x, \xi)),$$

$$P_{m-i}(t, x, \tau, \xi)=\sum_{j=0}^{m-i} t^{\sigma(i,j)}P_{ij}(t, x, \xi)\tau^{m-i-j},$$

where $D_t=\dfrac{\partial}{\sqrt{-1}\partial t}$, $D_x=(D_{x_1}, \cdots, D_{x_n})=\left(\dfrac{\partial}{\sqrt{-1}\partial x_1}, \cdots, \dfrac{\partial}{\sqrt{-1}\partial x_n}\right)$, $\ell \in \boldsymbol{N}$, $\sigma(i, j)=\max(j\ell-i, 0)$, $\lambda_i(t, x, \xi) \in C^\infty(\boldsymbol{R}_t \times \boldsymbol{R}_x^n \times \boldsymbol{R}_\xi^n \backslash 0, \boldsymbol{R}\backslash 0)$ are homogeneous of degree 1 with respect to ξ, and $P_{ij}(t, x, \xi) \in C^\infty(\boldsymbol{R}_t \times \boldsymbol{R}_x^n \times \boldsymbol{R}_\xi^n)$ are homogeneous polynomials of degree j with respect to ξ. Moreover, $\lambda_i(t, x, \xi)$ satisfy $|\lambda_i(t, x, \xi)-\lambda_j(t, x, \xi)| \geq C|\xi|$ $(t \in \boldsymbol{R}, x \in \boldsymbol{R}^n, \xi \in \boldsymbol{R}^n\backslash 0)$ for some $C>0$ if $i \neq j$.

As for P, Uryu [8] established the $\mathcal{E}$ wellposedness of the Cauchy problem and Nakamura-Uryu [4] and Shinkai [6] illustrated the construction of a backward and a forward parametrices of the Cauchy problem with initial data at $t=0$ in terms of Fourier integral operators.

In this note we show that the equation $Pu=0$ possesses a solution whose singularities branch at $t=0$. The outline of the proof is as follows. According to the construction of parametrix given by Nakamura-Uryu [4], the main parts of the amplitudes which consist the parametrix are determined by a fundamental system of solutions of the ordinary differential operator

$$L=\sum_{i=0}^{m} \sum_{j=0}^{m-i} t^{\sigma(i,j)}P_{ij}(0, x, \xi)D_t^{m-i-j}.$$

Its asymptotic expansions for large $|\xi|$ considered in $t>0$ and $t>0$ are different (namely, Stokes phenomena occurs at $t=0$). The one is different from the other by multiplying Stokes multipliers. Observing