34. *An Asymptotic Property of a Certain Brownian Motion Expectation for Large Time*

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1. Let $(X(t): t \geqq 0, P)$ be the Brownian motion in $\boldsymbol{R}^d$ starting from $X(0)=0$. We give an asymptotic formula for the quantity

$$(1) \quad J(t)=J(t;\varphi)=E\left[\exp\left\{-\nu\int_{\boldsymbol{R}^d}\left\{1-\exp\left(-\int_0^t \varphi(X(\sigma)-y)d\sigma\right)\right\}dy\right\}\right]$$

as $t\to\infty$, where E denotes the expectation with respect to P, φ a non-negative Borel function on $\boldsymbol{R}^d$ and $\nu>0$ a constant. Asymptotic behavior of $J(t)$ has been investigated in connection with the study of the spectral distributions of the Schrödinger operators $-1/2\Delta+q(x)$ with random potentials of the form $q(x)=\sum \varphi(x-\xi_n)$, where $\{\xi_n\}$ is the support of the Poisson random measure with intensity $\nu>0$ (see [2]–[7]).

Donsker and Varadhan [2] proved that if $\varphi(x)=o(1/|x|^{d+2})(|x|\to\infty)$ and $\int \varphi(x)dx>0$, then

$$(2) \quad \lim_{t\to\infty} t^{-d/(d+2)}\log J(t)=-k(\nu)$$

exists and

$$(3) \quad k(\nu)=\nu^{2/(d+2)}\frac{d+2}{2}(2\lambda_1/d)^{d/(d+2)},$$

where λ_1 is the smallest eigenvalue for $-1/2\Delta$ in a sphere of unit volume with zero boundary condition. On the other hand, Pastur [7] proved that if $\varphi(x)\sim K/|x|^{d+\beta}(|x|\to\infty)$, where $K>0$ and $0<\beta<2$, then

$$(4) \quad \lim_{t\to\infty} t^{-d/(d+\beta)}\log J(t)=-\kappa(\nu,\beta,K)$$

exists and

$$(5) \quad \kappa(\nu,\beta,K)=\nu K^{d/(d+\beta)}\Gamma\left(\frac{\beta}{d+\beta}\right)\Omega_d,$$

where Ω_d is the volume of a sphere of unit radius. The following theorem covers the critical case of $\varphi(x)\sim K/|x|^{d+2}(|x|\to\infty)$.

Theorem 1. *Let $(X(t), t\geqq 0)$ be the d-dimensional Brownian motion with $X(0)=0$. Suppose φ is a non-negative bounded Borel function of $\boldsymbol{R}^d$ such that $\varphi(x)\sim K/|x|^{d+2}(|x|\to\infty)$, where $K>0$. Define $J(t)$ by (1). Then for any $\nu>0$*

$$(6) \quad \lim_{t\to\infty} t^{-d/(d+2)}\log J(t)=-C(\nu,K)$$

exists and $C(\nu,K)=\inf_{f\in\mathcal{F}_0}[I(f)+\Phi(f)]$, where