31. *Congruences between Siegel Modular Forms of Degree Two. II*

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Introduction. We supplement the previous note [6] by describing liftings of congruences. In particular, the congruences in Theorems 2 and 3 of [6] are considered to be congruences lifted from degree 1 to degree 2. The author would like to thank Prof. H. Maass for communicating that Prof. D. Zagier ([16]) proved completely the Conjectures 1 and 2 of [5] by using recent results of Prof. W. Kohnen after Maass [10] [11] [12] and Andrianov [2] (cf. § 1 below).

§ 1. **Liftings.** We denote by $M_k(\Gamma_n)$ (resp. $S_k(\Gamma_n)$) the vector space over the complex number field $\boldsymbol{C}$ consisting of all Siegel modular (resp. cusp) forms of degree n and weight k for integers $n \geqq 0$ and $k \geqq 0$. The space of Eisenstein series is denoted by $E_k(\Gamma_n)$ which is the orthogonal complement of $S_k(\Gamma_n)$ in $M_k(\Gamma_n)$ with respect to the Petersson inner product. We say that a modular form f in $M_k(\Gamma_n)$ is eigen if f is a non-zero eigenfunction of all Hecke operators on $M_k(\Gamma_n)$. Let f be an eigen modular form in $M_k(\Gamma_n)$ for $n=1,2$. We define the (standard) Hecke polynomial at a prime p by $H_p(T,f)=1-\lambda(p,f)T+p^{k-1}T^2$ if $n=1$, and $H_p(T,f)=1-\lambda(p,f)T+(\lambda(p)^2-\lambda(p^2)-p^{2k-4})T^2-p^{2k-3}\lambda(p)T^3+p^{4k-6}T^4$ if $n=2$, where T is an indeterminate and $\lambda(m,f)$ is the eigenvalue of the Hecke operator $T(m)$ for $f: T(m)f=\lambda(m,f)f$. We define the (standard) L-function by $L(s,f)=\prod_p H_p(p^{-s},f)^{-1}$ where p runs over all prime numbers. We denote by $\boldsymbol{Q}(f)$ the field generated by $\{\lambda(m,f)\,|\,m \geqq 1\}$ over the rational number field $\boldsymbol{Q}$, and we put $\boldsymbol{Z}(f)=\boldsymbol{Q}(f)\cap\bar{\boldsymbol{Z}}$, where $\boldsymbol{Z}$ is the rational integer ring, and $\bar{\boldsymbol{Z}}$ is the ring of all algebraic integers in $\boldsymbol{C}$. Then $\boldsymbol{Q}(f)$ is a totally real finite extension of $\boldsymbol{Q}$, and $\boldsymbol{Z}(f)$ is the integer ring of $\boldsymbol{Q}(f)$. See [7] which contains the case of general degree.

We consider the following two liftings from degree 1 to degree 2 for each even integer $k \geqq 4$.

(A) The first lifting is the $\boldsymbol{C}$-linear injection $[\ ]: M_k(\Gamma_1)\to M_k(\Gamma_2)$ defined in [8] (cf. [6] [9]), which is given by the (generalized) Eisenstein series. For each eigen modular form f in $M_k(\Gamma_1)$ we have that: $[f]$ is an eigen modular form satisfying $H_p(T,[f])=H_p(T,f)H_p(p^{k-2}T,f)$ for all p and $L(s,[f])=L(s,f)L(s-k+2,f)$.

(B) The second lifting is the $\boldsymbol{C}$-linear injection σ_k: $M_{2k-2}(\Gamma_1)$