29. On the Regularity of Arithmetic Multiplicative Functions. II

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In our previous paper ([1]) we discussed some sufficient conditions under which an arithmetic multiplicative function turns out to be completely multiplicative. In this paper we shall extend and refine the previous results and make some remarks about remaining problems in this field.

1. Let S be a sequence in N of density zero, and $\mathcal{C}_S$ be the set of all those sequences $\{a_n\}_{n=1}^{\infty}$ which satisfy the condition

$$\lim_{x\to\infty}\frac{1}{x}\sum_{\substack{n\leqslant x\\ n\notin S}}|a_n|=0.$$

If $S=\phi$, we abbreviate $\mathcal{C}_S$ to $\mathcal{C}$.

Theorem. *Let $F(n)$ and $G(n)$ be arithmetic multiplicative functions, a and b be positive integers and $(a, b)=1$, ε be either $+1$ or -1, fixed arbitrary. Suppose $|F(an+\varepsilon b)|=1$ if n and $an+\varepsilon b \in N$, $|G(n)|=1$ for any $n\in N$, and $\{F(an+\varepsilon b)-C\cdot G(n)\}_{n=1}^{\infty}\in \mathcal{C}_S$ for some S and for some constant C.*

I) *When a is even, we can decompose $F(n)$ and $G(n)$:*

$$G(n)=G'(n)\cdot H(n) \qquad \textit{for any } n\in N,$$
$$F(n)=G'(n)\cdot H(n) \qquad \textit{for any } n \textit{ such that } (n, a)=1,$$

where $G'(n)$ and $H(n)$ are multiplicative functions satisfying

i) *$G'(n)$ is completely multiplicative,*

ii) *$H(n)=H((n, b))$ for any $n\in N$,*

iii) *$\{G'(kn+\varepsilon)-G'(k)\cdot G'(n)\}_{n=1}^{\infty}\in \mathcal{C}$, for any $k\geqslant a$.*

Further we have

$$C=G'(a).$$

II) *When a is odd, suppose $2^{\alpha}\| b$ $(\alpha\geqslant 0)$. We can decompose $F(n)$ and $G(n)$:*

$$G(n)=G'(n)\cdot H(n)\cdot H'_{\alpha}(n) \qquad \textit{for any } n\in N,$$
$$F(n)=G'(n)\cdot H(n)\cdot \overline{H'_{\alpha}(n)} \qquad \textit{for any } n \textit{ such that } (n, a)=1,$$

where $\bar{z}$ denotes the complex conjugate of z, $G'(n)$ and $H(n)$ are multiplicative functions satisfying the above i)–iii), *and $H'_{\alpha}(n)$ is the multiplicative function which is defined as follows;*

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