23. *Potential Operators Associated with Absorbing Bessel Processes*

By Junji TAKEUCHI
Department of Mathematics, Ochanomizu University

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1. Introduction. Let α be any real number. Consider a conservative Bessel diffusion process of index α on the half line $\boldsymbol{R}^+=[0, \infty)$ determined by infinitesimal generator

$$A=\frac{1}{2}\left(\frac{d^2}{dx^2}+\frac{\alpha-1}{x}\frac{d}{dx}\right). \tag{1.1}$$

Using the terminology for diffusions described in Itô-McKean [2, p. 130], here is the boundary classification:

(i) For all real α, the end point ∞ is a natural (not exit, not entrance) boundary;

(ii) $\alpha\geqq 2$: 0 is an entrance but not exit point;
$0<\alpha<2$: 0 is an entrance and exit point;
$\alpha\leqq 0$: 0 is an exit but not entrance point.

Particularly in the case $0<\alpha<2$, appropriate boundary condition must be imposed at the origin. The most familiar condition is the case of an instantaneously reflecting barrier. In a previous paper [1], Arakawa and the present author gave the domain and the representation of potential operator for the reflecting Bessel process. The potential kernel obtained there is expressible as $U(x\vee y)$, where

$$U(x)=\begin{cases}\dfrac{1}{\alpha-2}\cdot\dfrac{1}{x^{\alpha-2}} & \text{if } 0<\alpha<2 \text{ and } \alpha>2,\\ \log\dfrac{1}{x} & \text{if } \alpha=2\end{cases}$$

and $x\vee y$ denotes the greater of x and y.

In this note we deal with the Bessel process of index α $(-\infty<\alpha<2)$ with an absorbing barrier at the origin. More precisely, once a diffusion particle reaches at the origin, it stays there forever. It turns out that the quantity $x\wedge y$, i.e., the smaller of x and y appears in the potential kernels in common. This phenomenon forms a fine contrast to $x\vee y$ in the case of reflecting barrier. Another objective of ours is to study the Bessel process with negative index, for most investigations about Bessel processes have been restricted within the case of positive index.

Associated with the absorbing Bessel process is the infinitesimal generator A given by (1.1) acting on the domain