19. *On the Group of Units of a Non-Galois Quartic or Sextic Number Field*

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All number fields we consider are in the complex number field. The symbol $\langle S\rangle$ denotes a multiplicative group generated by S.

For a finite extension $k/\boldsymbol{Q}$, let E_k be the group of units of k, and E'_k be the group generated by all units of proper subfields of k together with roots of unity in k. We define the group H_k of relative units of k by

$H_k=\{\varepsilon\in E_k \mid N_{k/k'}(\varepsilon)$ is a root of unity for a proper subfield k' of $k\}$.

Let us consider the problem to construct E_k with the help of E'_k. It is interesting to utilize H_k together with E'_k when $(E_k:E'_k)=+\infty$. Hasse [2] has treated such a case when k is a real cyclic quartic number field. We are going to treat the case when k is a non-galois quartic (resp. sextic) number field having a quadratic subfield (resp. a quadratic and a cubic subfields). Then the galois closure of $k/\boldsymbol{Q}$ is a dihedral extension of degree 8 or 12 over $\boldsymbol{Q}$. We restrict our investigation on such extensions.

From now on, we assume $n=2$ or 3. Let $L/\boldsymbol{Q}$ be a galois extension of degree $4n$ with the galois group

$$G=\langle\sigma,\tau\rangle; \qquad \sigma^{2n}=\tau^2=(\sigma\tau)^2=1.$$

The invariant subfield of the subgroup $\langle\tau\rangle$ (resp. $\langle\sigma^3\tau\rangle$, $\langle\sigma^n\rangle$) is denoted by K (resp. F, Ω), and the maximal abelian subfield by Λ. Then K and F are non-galois number fields of degree $2n$ which we are going to study. The quadratic subfield of K (resp. F) is denoted by K_2 (resp. F_2). When $n=3$, the cubic subfield of both K and F is denoted by K_3. The quartic field Λ is the composite field of K_2 and F_2 which contains another quadratic subfield Λ_2. Note that $\Lambda=\Omega$ when $n=2$.

It is easy to show the following, which is in Nagell [6] when $n=2$.

Proposition 1. *When $L\cap\boldsymbol{R}=\Omega$, we have $E_K=E'_K$ and $E_F=E'_F$.*

Therefore we treat the two cases:

$$\text{Case I}: \ L\cap\boldsymbol{R}=K. \qquad \text{Case II}: \ L\subset\boldsymbol{R}.$$

Taking into account that all roots of unity of L is contained in the quartic subfield Λ, we take and fix a generator ω (resp. ζ, ρ) of the group of roots of unity of Λ (resp. Λ_2, F_2).

1. Type of E_K and E_F. A typical example of K and F are a pure number field of degree $2n$. The method, which is used in Stender [8],