18. *Block Intersection Numbers of Block Designs*

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§ 1. **Introduction.** In this note, we shall assume throughout that the block designs are nontrivial. For a t–(v, k, λ) design $\boldsymbol{D}$ we use λ_i $(0 \leqslant i \leqslant t)$ to represent the number of blocks which contain the given i points of $\boldsymbol{D}$. A t–(v, k, λ) design $\boldsymbol{D}$ is called block-schematic if the blocks of $\boldsymbol{D}$ form an association scheme with the relations determined by size of intersection (cf. [3]). For a block B of a t–(v, k, λ) design $\boldsymbol{D}$ we use $x_i(B)$ $(0 \leqslant i \leqslant k)$ to denote the number of blocks each of which has exactly i points in common with B. If $x_i(B)$ is uniquely determined by the choice of a block B for each i $(i=0, \cdots, k)$, we shall say that $\boldsymbol{D}$ is block-regular, and we shall write x_i instead of $x_i(B)$. We remark that if a t–(v, k, λ) design $\boldsymbol{D}$ is block-schematic, then $\boldsymbol{D}$ is block-regular. In this note, we shall give the following two theorems. The detailed proofs will be given elsewhere.

Theorem 1. *For each $n \geqslant 1$ and $\lambda \geqslant 1$,*

(a) *there exist at most finitely many block-schematic t–(v, k, λ) designs with $k-t=n$ and $t \geqslant 3$, and*

(b) *if also $\lambda \geqslant 2$, there exist at most finitely many block-schematic t–(v, k, λ) designs with $k-t=n$ and $t \geqslant 2$.*

Theorem 2. *Let c be a real number with $c>2$. Then for each $n \geqslant 1$ and $l \geqslant 0$, there exist at most finitely many block-regular t–(v, k, λ) designs with $k-t=n$, $v \geqslant ct$ and $x_i \leqslant l$ for some i $(0 \leqslant i \leqslant t-1)$.*

Remark. Since there exist infinitely many 2–$(v, 3, 1)$ designs, and since every 2–$(v, k, 1)$ design is block-schematic (cf. [2]), Theorem 1 does not hold for $\lambda=1$ and $t=2$.

§ 2. **Outline of the proof of Theorem 1.** Lemma 1. *Let $\boldsymbol{D}$ be a block-regular t–(v, k, λ) design. Then the following equation holds for $i=0, \cdots, k-1$.*

$$x_i=\sum_{j=i}^{t-1}\binom{j}{i}(\lambda_j-1)\binom{k}{j}(-1)^{i+j}+\sum_{j=t}^{k-1}\binom{j}{i}w_j(-1)^{i+j},$$

where $x_j \leqslant w_j \leqslant (\lambda-1)\binom{k}{j}$ $(t \leqslant j \leqslant k-1)$.

Lemma 2. *Let $\boldsymbol{D}$ be a t–(v, k, λ) design with $t, \lambda \geqslant 2$. If $v \geqslant k^3$, then there exist three blocks B_1, B_2, B_3 of $\boldsymbol{D}$ such that $|B_1 \cap B_2|=t-1$, $|B_2 \cap B_3| \geqslant t$ and $|B_1 \cap B_3|=t-2$.*

By making use of Lemmas 1, 2 and the idea of Atsumi [1], we get