14. *On a Representation of the Solution of the Cauchy Problem with Singular Initial Data*

By Tan ISHII
Department of Mathematics, Tokyo Metropolitan University

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1. Introduction. In this note we shall deal with the Cauchy problem with singular initial data in the complex domain for partial differential operators with holomorphic characteristic roots. This problem was introduced by Hamada [1] and developed by Hamada-Leray-Wagschal [2], Nakamura [4] and Hamada-Nakamura [5]. Recently, Kumano-go-Taniguchi [3] constructed the fundamental solution for a hyperbolic system in the real variables' case. Their form of the solution is suggestive to ours. In fact their multiphase function corresponds to our multicharacteristic function, and so does their infinite sum of iterated integral. However, our construction of the solution is quite different from theirs; we solve the transport equations.

2. Notations and result. Let $H(t, x; \partial_t, \partial_x)$ be a partial differential operator of order m whose coefficients are holomorphic in

$$\Omega=\{|t|+\textstyle\sum_{i=1}^n |x_i|<R\}\subset C^{n+1} \qquad (R>0),$$

and the coefficient of ∂_t^m is identically 1. We impose the following condition on the principal part $h(t, x; \partial_t, \partial_x)$ of $H(t, x; \partial_t, \partial_x)$:

(A) $h(t, x; \tau, \zeta)$ is written as $h(t, x; \tau, \zeta)=\prod_{j=1}^m (\tau+\lambda_j(t, x; \zeta))$, where each $\lambda_j(t, x; \zeta)$ is holomorphic in $\Omega\times\Omega^*=\Omega\times\{\sum_{i=1}^n |\zeta_i-\delta_{1,i}|<R_1\}$ $(R_1>0)$.

We shall consider the following Cauchy problem with singular data in Ω:

$$\text{(E)} \qquad H(t, x; \partial_t, \partial_x)u(t, x)=0;$$

$$\text{(C)} \qquad (\partial_t)^l u(0, x)=\frac{c_l(x')}{x_1^{p_l}} \qquad (l=0, 1, \cdots, m-1),$$

where $c_l(x')$ is holomorphic in $\{\sum_{i=2}^n |x_i|<R\}$, and p_l a positive integer.

We shall construct the solution $u(t, x)$ of (E), (C) in the form

$$\text{(U)} \qquad \begin{aligned} u(t, x)=&\textstyle\sum_{j=j_1}^\infty f_j(\phi_1(t, x))a_{j,1}(t, x)\\ &+\textstyle\sum_{k=2}^\infty \int_0^t dt_1\int_0^{t-t_1} dt_2\cdots\int_0^{t-t_1-\cdots-t_{k-2}} dt_{k-1}\\ &\times\textstyle\sum_{j=j_k}^\infty f_j(\phi_k((t)_k, x))a_{j,k}((t)_k, x). \end{aligned}$$

Here $(t)_k=(t_1, \cdots, t_k)\in C^k$, $t_1+\cdots+t_k=t$; for $s\in C$, $f_j(s)=(s^j/j!)(\log s-\Gamma'(j+1)/\Gamma(j+1))$ for $j\geq 0$ and $f_j(s)=(-1)^{j+1}(-j-1)!\, s^j$ for $j\leq -1$. $\phi_k((t)_k, x)$ and $a_{j,k}((t)_k, x)$ $(k\in N, j\in Z)$ are holomorphic at the origin of $C^k\times C^n$. The $(k-1)$-*ple* integral is the iterated integral along the seg-