# 113. *Analyticity of Complements of Complete Kähler Domains**)

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§1. **Statement of the result.** Let $A$ be a real submanifold of a complex manifold $M$. We want to know the conditions on $X:=M-A$ which force $A$ to be a complex submanifold of $M$. Our main result is the following

**Theorem.** *Under the above notations, assume that*

1) *$X$ has a complete Kähler metric*

*and that*

2) *$A$ is a regular submanifold of class $C^1$ with real codimension* 2. *Then $A$ is a complex submanifold of $M$.*

Our theorem amounts to a partial answer to the following problem which was asked by T. Nishino.

**Problem.** Let $D\subset C^n$ be a domain and $f:D\to C$ a continuous function. Assume that there exists a plurisubharmonic function $\varphi$ on a neighbourhood of $G(f):=\{(z', f(z'))\,;\, z'\in D\}$ such that $G(f)=\{z\,;\,\varphi(z)=-\infty\}$. *Is $G(f)$ a complex submanifold of $D\times C$?*

I express sincere thanks to Dr. Y. Nishimura, who told me the problem and encouraged me.

§2. **Proof of the theorem.** Let $X$ be a complex manifold of dimension $n$. $X$ is called a complete Kähler manifold if $X$ has a complete Kähler metric, i.e., a Kähler metric (of class $C^2$) which makes $X$ a complete metric space.

**Proposition** (cf. Corollary (1.7) in [1]). *Let $X$ be a complete Kähler manifold, $\varphi$ a bounded strictly plurisubharmonic function of class $C^4$ on $X$ and $f$ a measurable $(n,1)$-form on $X$. Assume that $f$ is square integrable with respect to the metric*

$$ds^2:=\sum_{\alpha,\beta}\frac{\partial^2\varphi}{\partial z^\alpha\partial\bar z^\beta}dz^\alpha d\bar z^\beta,$$

*where $(z^1,\cdots,z^n)$ denotes a local coordinate of $X$. Then there exists a square integrable $(n,0)$-form $g$ on $X$ satisfying $\bar\partial g=f$ if and only if $\bar\partial f=0$.*

Let $M$ be a complex manifold containing $X$ as a domain. We assume that $A:=M-X$ is a real two codimensional regular submanifold

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