109. *Riemann-Lebesgue Lemma for Real Reductive Groups*

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1. Introduction. Let G be a Lie group of class $\mathcal{H}$, which is a reductive group defined in §2. Let $P=MAN$ be a cuspidal parabolic subgroup of G and its Langlands decomposition. For any representation σ of discrete series of M and (not necessarily unitary) character λ of A, we can associate a continuous representation $\pi_{\sigma,\lambda}^{(P)}$ of G. The Fourier-Laplace transform of $f \in C_c(G)$ is defined by

$$\hat{f}_P(\sigma, \lambda)=\int_G f(x)\pi_{\sigma,\lambda}^{(P)}(x)\,dx.$$

Let V_σ be the representation space of σ. Let K be a maximal compact subgroup of G. Then $\hat{f}_P(\sigma, \lambda)$ is an integral operator on a subspace $\mathfrak{H}_\sigma$ of $L^2(K; V_\sigma)$ with the kernel function $\hat{f}_P(\sigma, \lambda; k_1, k_2)$, $k_1, k_2 \in K$. If λ is unitary, $\hat{f}_P(\sigma, \lambda)$ is defined for $L^1(G)$ and it vanishes when $(\sigma, \lambda)\to\infty$ in the sense of hull-kernel topology (see [2, p. 317]). The purpose of the present paper is to show that there exists a tube domain $\mathcal{F}^1$, containing the unitary dual A^* of A, of the complexification of A^* such that for almost all $(k_1, k_2) \in K\times K$ $\hat{f}_P(\sigma, \lambda; k_1, k_2)$ is defined for $f \in L^1(G)$ and it vanishes when $\lambda=\xi+i\eta \in \mathcal{F}^1$ and $(\sigma, \lambda)\to\infty$.

2. Notation and preliminaries. If V is a real vector space, V_c denotes its complexification. Let G be a Lie group with Lie algebra $\mathfrak{g}$. Let G^0 be the connected component of the unit of G. We denote by G_1 the analytic subgroup of G whose Lie algebra is $\mathfrak{g}_1=[\mathfrak{g}, \mathfrak{g}]$. Let G_c be the connected complex adjoint group of $\mathfrak{g}_c$. A Lie group G with Lie algebra $\mathfrak{g}$ is called of class $\mathcal{H}$ if G satisfies the following conditions: (1) $\mathfrak{g}$ is reductive and $\mathrm{Ad}(G)\subset G_c$; (2) the center of G_1 is finite; (3) $[G:G^0]<\infty$. In the sequel, we assume that G is a Lie group of class $\mathcal{H}$. If L is a Lie group, we denote by $\mathfrak{l}=\mathrm{LA}(L)$ the Lie algebra of L.

Let K be a maximal compact subgroup of G. Let $\mathfrak{g}=\mathfrak{k}\oplus\mathfrak{s}$, $\mathfrak{k}=\mathrm{LA}(K)$, be the Cartan decomposition of $\mathfrak{g}$ and θ the corresponding Cartan involution. Let $\mathfrak{a}_0$ be a maximal abelian subspace of $\mathfrak{s}$ and $\mathfrak{a}_0^*$ its dual space. We denote by Δ the set of all roots of $(\mathfrak{g}, \mathfrak{a}_0)$. For $\alpha \in \Delta$, let $\mathfrak{g}_\alpha$ be the corresponding root space. We fix an order in $\mathfrak{a}_0^*$ and denote by Δ^+ the set of all positive roots. We set $\mathfrak{n}_0=\sum_{\alpha\in\Delta}\oplus\mathfrak{g}_\alpha$. Let M_0 be the centralizer of $\mathfrak{a}_0$ in K. We put $A_0=\exp\mathfrak{a}_0$, $N_0=\exp\mathfrak{n}_0$

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