107. *Surgery of Domain and the Green's Function of the Laplacian*

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§1. **Introduction and results.** Let M be a bounded domain in $\mathbf{R}^m$ with smooth boundary. Let N be a compact connected regular smooth submanifold of M. We put $n=\dim N$. In this note we assume that $m\geq n+2\geq 3$. For any sufficiently small $\varepsilon>0$, let Γ_ε be the ε-tubular neighbourhood of N defined by

$$\Gamma_\varepsilon=\{x\in M;\ \mathrm{dist}(x, N)<\varepsilon\}.$$

We put $M_\varepsilon=M\backslash\bar{\Gamma}_\varepsilon$.

Let $G(x, y)$ (resp. $G_\varepsilon(x, y)$, $\varepsilon>0$) be the Green's function of the Laplacian with the Dirichlet condition on ∂M (resp. ∂M_ε).

In this paper we report the following theorems.

Theorem 1. *Assume $m-n\geq 5$, then for any fixed $x, y\in M\backslash N$*

$$G_\varepsilon(x, y)=G(x, y)-3S_5\varepsilon^{m-n-2}\int_N G(x, w)G(y, w)dw+O(\varepsilon^{m-n})$$

holds when ε tends to zero. Here S_k denotes the area of the unit sphere in $\boldsymbol{R}^k$.

Theorem 2. *Assume $m-n=3$ or 4, then for any fixed $x, y\in M\backslash N$*

$$G_\varepsilon(x, y)=G(x, y)-(m-n-2)S_{m-n}\varepsilon^{m-n-2}\int_N G(x, w)G(y, w)dw+O(K(\varepsilon))$$

holds when ε tends to zero. Here $K(\varepsilon)=\varepsilon^4|\log\varepsilon|$ in case $m-n=4$ and $K(\varepsilon)=\varepsilon^2$ in case $m-n=3$.

Theorem 3. *Assume $m-n=2$, then for any fixed $x, y\in M\backslash N$*

$$G_\varepsilon(x, y)=G(x, y)+(2\pi)(\log\varepsilon)^{-1}\int_N G(x, w)G(y, w)dw+O((\log\varepsilon)^{-2})$$

holds when ε tends to zero.

It should be remarked that the remainder terms $O(\varepsilon^{m-n})$, $O(K(\varepsilon))$, $O((\log\varepsilon)^{-2})$ in Theorems 1–3 are not uniform with respect to x, y.

Theorems 1–3 above are the versions of the Schiffer-Spencer formula which describes the asymptotic property of $G_\varepsilon(x, y)$ when ε tends to zero in case $m=2$ and $n=0$. See Schiffer-Spencer [2]. The author considers the case $m\geq 3$, $n=0$ in Ozawa [1].

Using the techniques developed in [1], we can get an asymptotic formula for eigenvalues of the Laplacian. We give some notations. Let $0>\lambda_1(\varepsilon)\geq\lambda_2(\varepsilon)\geq\cdots$ be the eigenvalues of the Laplacian in M_ε with the Dirichlet condition on ∂M_ε. And let $0>\lambda_1\geq\lambda_2\geq\cdots$ be the eigen-