98. On a Result of T. Watanabe on Excessive Functions

By L. STOICA

Department of Mathematics, National Institute for Scientific and Technical Creation, Bucharest, Romania

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Let $(\Omega, \mathcal{M}, \mathcal{M}_t, X_t, \theta_t, P^x)$ be a standard process with state space E (locally compact, denumerable base) and suppose that its resolvent $\{V_\lambda : \lambda > 0\}$ has the following property:

$$V_\lambda(C_b(E)) \subset C_b(E) \qquad \text{for each } \lambda > 0,$$

where $C_b(E)$ is the space of all bounded continuous functions on E.

The aim of this note is to prove the following result, which extends and unifies two results of T. Watanabe (Theorems 1 and 2 in [5]):

Theorem. *Let $f: E \to [0, \infty]$ be a lower semicontinuous function. Assume that for each $x \in E$ there exists a family of nearly Borel sets $\mathcal{U}(x)$ such that*

1° *$\mathcal{U}(x)$ is a base of neighbourhoods of x,*

2° *$E^x(f(X_{T_{CU}})) \leqslant f(x)$ for each $U \in \mathcal{U}(x)$.*

Then f is an excessive function.

The proof makes use of Bauer's minimum principle. We also need the following consequence of a result of G. Mokobodzki:

Lemma. *If the potential kernel V_0 maps the space of all continuous functions with compact support $C_c(E)$ into $C_b(E)$, then for each $g \in C_{c+}(E)$,*

$$\inf\{t : t \text{ is a lower semicontinuous excessive function and } t \geqslant V_0\, g \text{ on } CK, \text{ for some compact set } K\} = 0$$

Proof. From Theorem 12, p. 231 of [3], we deduce for each lower semicontinuous function g, the function Rg defined by

$$Rg = \inf\{t : t \text{ is an excessive function and } t \geqslant g\}$$

is a lower semicontinuous excessive function. (It should be noted that in [3] are considered only Borel excessive functions but the methods work for universally measurable functions.) Therefore if $g \in C_c^+(E)$ and K is a compact set, then $R(\chi_{CK} V_0 g)$ is lower semicontinuous. From Hunt's theorem (see [2], page 141) we know that $R(\chi_{CK} V_0 g)(x) = E^x(V_0 g(X_{T_{CK}})) = E^x\left(\int_{T_{CK}}^{\infty} g(X_t)dt\right)$ and hence $R(\chi_{CK} V_0 g) \to 0$ when $K \nearrow E$, which implies the lemma.

Proof of the theorem. In order to simplify the exposition we first assume that the potential kernel V_0 has also the property $V_0(C_b(E)) \subset C_b(E)$. Next we are going to prove $\lambda V_\lambda f \leqslant f$, for $\lambda > 0$. Since f