90. *On the Linear Sieve. II*

By Yoichi MOTOHASHI
Department of Mathematics, College of Science
and Technology, Nihon University

(Communicated by Kunihiko KODAIRA, M. J. A., Oct. 13, 1980)

1. The purpose of the present note is to show briefly an alternative proof of Iwaniec's remarkable improvement [2] upon the linear sieve of Rosser. Our argument is not much different from Iwaniec's, but, being a straightforward refinement of [3], it is comparatively more direct and easy. Roughly speaking, our procedure is an injection of a smoothing device to Rosser's infinite iteration of the Buchstab identity.

We retain most of the notations of [3], and in addition we introduce the condition Ω_∞: For any $3\le u<v$

$$\sum_{u\le p<v}\delta(p^2)p^{-2}=O((\log\log u)^{-1}).$$

Then the linear sieve of Iwaniec is, in a modified form,

Theorem. *Provided* $MN\ge z^2$, Ω_∞, $\Omega_2(1,L)$, $L\le(\log z)/(\log\log z)$, *we have, for* $\nu=0$ *and* 1,

$$(-1)^{\nu-1}\left\{S(A,z)-\left(\phi_\nu\left(\frac{\log MN}{\log z}\right)+O((\log\log z)^{-1/50})\right)XV(z)\right\}$$
$$\le\log z\,\mathop{\mathrm{Max}}_{\alpha,\beta}\left|\sum_{\substack{m<M\\ n<N}}\alpha_m\beta_nR_{mn}\right|,$$

where $\{\alpha_m\}$, $\{\beta_n\}$ *are variable vectors such that* $|\alpha_m|\le 1$, $|\beta_n|\le 1$.
Detailed discussions will be given in [5], and here we indicate only the clues. Note that we have obtained a hybrid of this result (but $\nu=1$ only) with the multiplicative large sieve (see [4]).

Acknowledgement. The present author is very much indebted to his friend Dr. H. Iwaniec for sending him a preprint of the monumental work [2].

2. To state our principal lemmas we introduce the following conventions: We put $z=z_1z_2^J$, where J is a large integer to be specified later. We dissect $[z_1,z)$ into J smaller intervals $[z_1z_2^{j-1},z_1z_2^j)$, and denote one of them generally by I with or without suffix. K with or without suffix stands for the set-theoretic direct product of a sequence of I's, and $\omega(K)$ be the number of constituent I's. If $K=I_1I_2\cdots I_r$ then $I<K$ means that $(I)<\min(I_j)$ $(j\le r)$, where (I) is the right end point of I; also $d\in K$ implies that $d=p_1p_2\cdots p_r$ with $p_j\in I_j$, $p_j\in P$. Note that we do not reject non-squarefree d. Next we define Φ_ν and Γ_ν $(\nu=0,1)$ to be the characteristic functions of the sets of K such that