77. *Cross Ratios as Moduli of del Pezzo Surfaces of Degree One*

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1. The projective plane with eight points on it blown up is called a del Pezzo surface of degree 1 provided the points are in general position, Manin [2]. For any such surface the linear system of anti-canonical divisors has exactly one fixed point, and if one further blows it up, then one obtains a rational elliptic surface with only irreducible fibers. According to Kodaira [1], the global sections of such an elliptic surface are exactly the exceptional curves of the first kind. Suppose conversely that there is given a rational elliptic surface with only irreducible fibers. Then, if one blows down one of the global sections, one obtains a del Pezzo surface of degree 1, and the isomorphism class of this del Pezzo surface does not depend on the choice of the global section to be blown down. We therefore prefer to study rational elliptic surfaces rather than del Pezzo surface of degree 1; for, much is known about the former.

Now let S be a rational elliptic surface with only irreducible fibers. Then the Néron-Severi group $N(S)$ (=the second homology group) of S is generated by the classes of global sections and the canonical class K; Shioda [4]. (The fibers belong to $-K$; i.e., they are the anti-canonical divisors of S.) Since $K^2=0$, the orthogonal complement $K^{\perp}=\{D \in N(S)\,;\, D\cdot K=0\}$ contains the multiples $\{K\}$ of K. We set now

$$\Gamma(S)=K^{\perp}/\{K\}$$

and call it the module associated with S. $\Gamma(S)$ is free $\boldsymbol{Z}$-module of rank 8 and the intersection form induces a negative definite, even, integral quadratic form of determinant 1 on $\Gamma(S)$. Therefore there are isometries between $\Gamma(S)$ and the module Γ of weights of E_8 with respect to this induced form and the Killing form. The surface S endowed with an isometry α of Γ onto $\Gamma(S)$ is called a *marked rational elliptic surface* or simply an *MRE surface*, and is denoted by (S,α). For an MRE surface, the fibers are assumed to be irreducible throughout this note. Two MRE surfaces (S,α), (S',α') are called *isomorphic* if there is a biregular morphism β of S onto S' such that $\beta_*\alpha=\alpha'$ where β_* denotes the isometry of $\Gamma(S)$ onto $\Gamma(S')$ induced by β. The purpose of this note is to explain briefly how to describe the set $\mathcal{M}$ of isomorphism classes of MRE surfaces as an algebraic variety.