74. $\mathcal{E}$-Well Posedness of Mixed Initial-Boundary Value Problem with Constant Coefficients in a Quarter Space. II

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1. **Introduction.** In this paper, we consider the following mixed problem:

(1.1) $A(D)u(t, x, y)=f(t, x, y), \quad (t, x, y) \in X,$

(1.2) $B_j(D)u(t, x, y)|_{x=0}=g_j(t, x, y), \quad j=1, \cdots, q, \quad (t, y) \in \boldsymbol{R}^{n+1},$

(1.3) $u(t, x, y)=0, \quad (t, x, y) \in X, \quad t \leqslant 0,$

where $X=\{(t, x, y) \in \boldsymbol{R}^{n+2};\ x>0\}$, $D=(D_t, D_x, D_y)=(-\sqrt{-1}\partial/\partial t, -\sqrt{-1}\partial/\partial x, -\sqrt{-1}\partial/\partial y)$ and $A(D)$, $B_j(D)$, $j=1, \cdots, q$, are differential operators with constant coefficients.

Definition 1.1. The mixed problem (1.1)–(1.3) is said to be $\mathcal{E}$-well posed if for arbitrary data $f \in C^\infty(\bar{X})$, $g_j \in C^\infty(\boldsymbol{R}^{n+1})$ such that $f=0$, $g_j=0$ when $t \leqslant 0$, there exists one and only one solution $u \in C^\infty(\bar{X})$ of the system of the equations (1.1)–(1.3).

Remark 1.2. According to the definition of $\mathcal{E}$-well posedness, we have that if data vanish when $t \leqslant T (T \geqslant 0)$, then the solution also vanishes when $t \leqslant T$. From this point of view, we can consider the variable t as time.

Sakamoto [3] and the author [5] gave a necessary and sufficient condition for $\mathcal{E}$-well posedness of the mixed problem (1.1)–(1.3) under the assumption that $A^0(1, 0, 0) \neq 0$ (A^0 is the principal part of A). The purpose of this paper is to give a necessary and sufficient condition for $\mathcal{E}$-well posedness of the mixed problem (1.1)–(1.3) under the following assumption:

Assumption 1.3. $A^0(\tau, \zeta, 0) \not\equiv 0$.

Recently Nishitani [2] has studied the half space case under the same assumption as in Assumption 1.3. The author was stimulated by his work. The author would like to express his sincere gratitude to Prof. T. Nishitani, and also to Profs. M. Matsumura and S. Wakabayashi for their valuable advice.

2. **Hyperbolicity of the operator A.** In view of the closed graph theorem, we have that if the mixed problem (1.1)–(1.3) is $\mathcal{E}$-well posed, then there are positive constants T, Y, Z, C and an integer N such that

(2.1) $$|u(1,1,0)| \leqslant C\Big\{\sum_{j+k+|\alpha| \leqslant N} \sup_{0 \leqslant x \leqslant Z, |y| \leqslant Y, 0 \leqslant t \leqslant T} |D_t^j D_x^k D_y^\alpha A(D)u|$$