67. *Nonexistence of Minimizing Harmonic Maps from 2-Spheres*

By Akito FUTAKI
Department of Mathematics, University of Tokyo

(Communicated by Kunihiko KODAIRA, M. J. A., June 12, 1980)

§1. **Introduction.** Let (M, g) and (N, h) be compact Riemannian manifolds and $C^\infty(N, M)$ be the space of all smooth maps from N to M with the C^∞ topology. For $f \in C^\infty(N, M)$ we define its *energy* $E(f)$ by

$$E(f)=\frac{1}{2}\int_N h^{ij}\frac{\partial f^\alpha}{\partial x^i}\frac{\partial f^\beta}{\partial x^j}g_{\alpha\beta}*1. \tag{1.1}$$

A *harmonic map* is, by definition, a critical point of the functional E. A harmonic map is said to be *minimizing* if it minimizes energy in its connected component of $C^\infty(N, M)$, i.e. in its homotopy class.

When $\dim N=1$, N is a circle S^1 and a harmonic map $f: S^1 \to M$ is a closed geodesic. It is well known that every component of $C^\infty(S^1, M)$ contains a minimizing closed geodesic. In contrast with this, when $\dim N=2$, it is not always true that there exists a minimizing harmonic map in each component of $C^\infty(N, M)$. For instance there exists no minimizing harmonic map of degree ± 1 from a Riemann surface of genus $\geqq 1$ to a Riemann sphere whatever metrics are chosen on them ([5]).

On the other hand, Sacks and Uhlenbeck [8] established an existence result when $N=S^2$. Their result was applied to the proof of Frankel's conjecture by Siu and Yau [9] and to the study on the topology of 3-manifolds by Meeks and Yau [7]. The following is a result of Sacks and Uhlenbeck refined by Siu and Yau. Let M be a compact 1-connected Riemannian manifold. Let $f_0 \in C^\infty(S^2, M)$. Then there exist minimizing harmonic maps $f_1, \cdots, f_k \in C^\infty(S^2, M)$ such that $\sum_{i=1}^k f_i = f_0$ in $\pi_2(M)$ and that

$$\sum_{i=1}^k E(f_i)=\inf\left\{\sum_{i=1}^p E(g_i) \,|\, p \in N,\ \sum_{i=1}^p g_i = f_0 \text{ in } \pi_2(M)\right\}. \tag{1.2}$$

However it has been unknown whether one can always find a single minimizing harmonic map homotopic to f_0 or not.

The purpose of this paper is to give a Riemannian manifold M and a component of $C^\infty(S^2, M)$ such that no minimizing harmonic map exists in this component.

§2. **Statement of the result. Theorem.** *Let M be a compact 1-connected Kähler surface. Suppose there are two disjoint rational*