56. *Analytic Expressions of Unstable Manifolds*

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§ 0. **Introduction.** In the study of bifurcations of differentiable dynamical systems, topological configurations of stable manifolds and unstable manifolds play an important role. In this note we give global analytic expressions by analytic mappings for unstable sets of strictly hyperbolic fixed points of analytic mappings $f: R^n \to R^n$. If mapping f is a diffeomorphism, the obtained unstable set agrees with the unstable manifold of the fixed point.

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§ 1. **Main theorems.** Let $f: R^n \to R^n$ be a real analytic map defined globally on R^n. We assume that the origin, O, is a fixed point of f, i.e., $f(O)=O$, and that the Jacobian matrix df_O at O is diagonalisable.

Let $\alpha_1, \alpha_2, \cdots, \alpha_n$ denote the eigenvalues of df_O. We assume O is hyperbolic, i.e.,

$$(1) \qquad \begin{cases} |\alpha_i|>1 & \text{for } i=1, 2, \cdots, k, \\ |\alpha_i|<1 & \text{for } i=k+1, \cdots, n. \end{cases}$$

Let $\delta=(\delta_1, \cdots, \delta_k)$ be multi-index with $\delta_i \geqq 0$ for $i=1, \cdots, k$. Let $\alpha=(\alpha_1, \cdots, \alpha_k)$. We denote $|\delta|=\delta_1+\delta_2+\cdots+\delta_k$ and $\alpha^\delta=\alpha_1^{\delta_1}\cdot\alpha_2^{\delta_2}\cdot\cdots\cdot\alpha_k^{\delta_k}$. We assume also

$$(2) \qquad \alpha^\delta \neq \alpha_i$$

for any multi-index δ with $|\delta| \geqq 2$ and $i=1, \cdots, k$.

Let E^u denote the subspace of tangent space T_0R^n spanned by the eigenvectors for eigenvalues $\alpha_1, \cdots, \alpha_k$. Space E^u is invariant under the differential map $df_O: T_OR^n \to T_OR^n$. Let $\eta: E^u \to E^u$ be the differential map df_o restricted on E^u, i.e.,

$$\eta(\xi)=df_0(\xi) \qquad \text{for } \xi \in E^u.$$

We call a point P in R^n an unstable point of o if there is a sequence of points $P_i \in R^n$, $i=0, -1, -2, \cdots$, such that $P_i=f(P_{i-1})$ for $i=0, -1, -2, \cdots$, $P=P_0$ and that P_i tends to the origin as i tends to $-\infty$. We denote the set of unstable points of O by W^u. We call W^u the unstable set of O. If f is a diffeomorphism, then W^u is nothing but the unstable manifold of O.

Theorem 1. *Let $f: R^n \to R^n$ be a real analytic map defined globally*