# 54. On Topological Characterizations of Complex Projective Spaces and Affine Linear Spaces

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(Communicated by Kunihiko KODAIRA, M. J. A., May 12, 1980)

In § 1 we present several conjectures. In § 2 we give partial answers to them. In § 3 we discuss remaining problems.

**§ 1. Conjectures.** **Conjecture ($A_n$).** *Let $U$ be a complex manifold of dimension $n$ with the homotopy type of a point. Suppose that there is a Kähler smooth compactification $M$ of $U$ such that $D=M-U$ is a smooth divisor on $M$. Then $U$ is isomorphic to an affine linear space $A^n$.*

**Remark 1.** The smoothness of $D$ is the essential assumption. Without it, $U$ need not be $A^n$ (see [12]).

In § 2 we reduce ($A_n$) to the following

**Conjecture ($B_n$).** *Let $M$ be a compact complex manifold with $\dim M=n$ and let $D$ be a smooth ample divisor on $M$. Suppose that the natural homomorphism $H_p(D; Z)\to H_p(M; Z)$ is bijective for $0\leqq p\leqq 2n-2$. Then $M\cong P^n$ and $D$ is a hyperplane section on it.*

**Remark 2.** An affirmative answer to ($B_n$) would solve the question of [5] (4.15) and give a sharpened form of Proposition V in [13]. See also § 2, Corollary 3.

In § 2 we reduce ($B_n$) to the following

**Conjecture ($C_n$).** *Let $M$ be a projective complex manifold such that the cohomology ring $H^{\cdot}(M; Z)$ is isomorphic to $H^{\cdot}(P^n; Z)\cong Z[x]/(x^{n+1})$. Suppose further that $c_1(M)$ is positive. Then $M\cong P^n$.*

**Remark 3.** It is well known that any projective manifold homeomorphic to $P^n$ is holomorphically isomorphic to $P^n$, provided that $c_1$ is positive. Moreover, the positivity assumption on $c_1$ is not necessary if $n$ is odd (see [8] and [11]). The proof depends on the theory of Pontrjagin classes.

**Remark 4.** ($C_n$) would not be true without the assumption on the ring structure. Indeed, any odd dimensional hyperquadric has a cohomology group isomorphic to that of $P^n$.

**§ 2. Partial answers.** **Theorem 1.** *Conjecture $C_n$ is true for $n\leqq 5$.*

We give an outline of our proof for the case $n=5$. In view of the isomorphism $H^{\cdot}(M; Z)\cong H^{\cdot}(P^n; Z)$, we regard the Chern classes