# 51. *On the Determination of all NB-Structures on BCK-Algebras*

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In this Note, we shall show that all $NB$-structures on a $BCK$-algebra are completely determined by a simple way, and the $NB$-structures give some surprising simplifications of complicated conditions which define various classes of $BCK$-algebras. Thus the $NB$-structures on a $BCK$-algebra may be considered as an auxiliary apparatus.

The $NB$-structure on a $BCK$-algebra was independently introduced by the present author and H. Rasiowa (see [1], [3]). To define it, we first recall a definition of $BCK$-algebras and its basic properties (for detail, see [2]).

A *BCK-algebra* $\langle X; *, 0\rangle$ is an algebra of type $\langle 2, 0\rangle$ satisfying the following conditions (1)–(5).

(1) $((x*y)*(x*z))*(z*y)=0$,

(2) $(x*(x*y))*y=0$,

(3) $x*x=0$,

(4) $0*x=0$,

(5) $x*y=y*x=0$ implies $x=y$.

If we define $x\leq y$ by $x*y=0$, then $X$ is a partially ordered set with respect to $\leq$.

For elements $x, y, z$ in a $BCK$-algebra;

(6) $x*0=x$,

(7) $(x*y)*z=(x*z)*y$.

If a $BCK$-algebra $X$ has a greatest element with respect to $\leq$, then $X$ is called to be *bounded*. The greatest element is denoted by 1.

If we define $Nx$ by $1*x$, then the following relations hold:

(8) $N0=1$, $N1=0$,

(9) $Nx*y=Ny*x$ for any $x, y$.

Generalizing this notion, we arrive at the notion of an $NB$-algebra.

If a unary operation $\sim$ on a $BCK$-algebra $X$ satisfies

(10) $\sim x*y=\sim y*x$

for any $x, y\in X$, then $X$ is called an *NB-algebra*.

Let $X$ be an $NB$-algebra. (10) implies

$$\sim x*0=\sim 0*x.$$

By (6), it follows that

(11) $\sim x=\sim 0*x$.