48. *Branching of Singularities for Degenerate Hyperbolic Operators and Stokes Phenomena*

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In terms of Fourier integral operators we give an explicit representation of solutions of the Cauchy problem for a certain class of degenerate hyperbolic equations, and determine precisely whether or not the solutions possess branching singularities. Our results reveal a close connection between branching of singularities and Stokes phenomena.

Alinhac [1] and Taniguchi-Tozaki [5] studied the problem of branching singularities for a special class of operators $\frac{\partial^2}{\partial t^2}-t^{2l}\frac{\partial^2}{\partial x^2}+$ (lower order terms) in $\boldsymbol{R}_t\times\boldsymbol{R}_x$. We study the same problem for the following type of higher order operators $P(t,D_t,D_x)$ in $\boldsymbol{R}_t\times\boldsymbol{R}_x^n$:

$$P(t,D_t,D_x)=\sum_{i=0}^{m}P_{m-i}(t,D_t,D_x),$$

$$P_{m-i}(t,D_t,D_x)=\sum_{j=0}^{m-i}t^{jl-i}p_{i,j}(D_x)D_t^{m-i-j},\qquad 0\leqq i\leqq m,$$

$$P_m(t,\tau,\xi)=\prod_{i=1}^{m}(\tau-t^l\lambda_i(\xi)),$$

where $D_t=\frac{1}{\sqrt{-1}}\frac{\partial}{\partial t}$, $D_x=(D_{x_1},\cdots,D_{x_n})=\left(\frac{1}{\sqrt{-1}}\frac{\partial}{\partial x_1},\cdots,\frac{1}{\sqrt{-1}}\frac{\partial}{\partial x_n}\right)$, $l\in\boldsymbol{N}$, $p_{i,j}(\xi)\in C^\infty(\boldsymbol{R}_\xi^n)$ are homogeneous of degree j with respect to ξ, $p_{i,j}(\xi)\equiv 0$ if $jl-i<0$ and $\lambda_i(\xi)\in C^\infty(\boldsymbol{R}_\xi^n\backslash 0,\boldsymbol{R}\backslash 0)$ are distinct. Here $C^\infty(\boldsymbol{R}_\xi^n\backslash 0,\boldsymbol{R}\backslash 0)$ denotes the set of all $\boldsymbol{R}\backslash 0$-valued C^∞ functions defined in $\boldsymbol{R}_\xi^n\backslash 0$.

Lemma 1. *If we introduce a new independent variable* $z=\frac{t^{l+1}}{l+1}|\xi|$, *then the ordinary differential operator* $P\left(t,\frac{1}{\sqrt{-1}}\frac{d}{dt},\xi\right)$ *is written as*

$$P\left(t,\frac{1}{\sqrt{-1}}\frac{d}{dt},\xi\right)=\sqrt{-1}^{-m}(l+1)^{lm/(l+1)}|\xi|^{m/(l+1)}z^{-m/(l+1)}L\left(z,\frac{d}{dz},|\xi|^{-1}\xi\right)$$

for $\xi\in\boldsymbol{R}_\xi^n\backslash 0$, *where*

$$L\left(z,\frac{d}{dz},\theta\right)=\sum_{i=0}^{m}\left(\sum_{j=0}^{i}a_{i,j}(\theta)z^j\right)z^{m-i}\frac{d^{m-i}}{dz^{m-i}},$$

$$a_{i,j}(\theta)=\sum_{j\leqq k\leqq h\leqq i}\sqrt{-1}^k(l+1)^{j-h}\alpha_1(m-k,m-h)\alpha_2(m-h,m-i)p_{k-j,j}(\theta)$$ *for* $\theta\in S_\theta^{n-1}$ *and*