45. A Remark on Ribet's Theorem

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Introduction. Let p be an odd prime, ζ_p be a primitive p-th root of unity and A be the p-Sylow subgroup of the ideal class group of $\boldsymbol{Q}(\zeta_p)$. In [5], Ribet obtained a remarkable theorem on the structure of A as a Galois module by means of modular forms. We obtain a generalization of this Ribet's Theorem.

After this work had been finished, Prof. M. Koike kindly informed the auther that he had obtained a result on the existence of modular forms satisfying a certain congruence (Koike [8]). By using his decisive result, he obtained a desirable generalization of our theorem.

Notations. For a prime p, let $\bar{\boldsymbol{Q}}_p$ (resp. $\bar{\boldsymbol{Q}}$) be an algebraic closure of $\boldsymbol{Q}_p$ (resp. $\boldsymbol{Q}$) and fix them. We fix embeddings $\bar{\boldsymbol{Q}}\to\boldsymbol{C}$ and $\bar{\boldsymbol{Q}}\to\bar{\boldsymbol{Q}}_p$, through which we regard elements of $\bar{\boldsymbol{Q}}$ as elements of $\boldsymbol{C}$ or $\bar{\boldsymbol{Q}}_p$. Let $\mathfrak{p}$ be the prime of $\bar{\boldsymbol{Q}}$, lying above p, corresponding to the fixed embedding $\bar{\boldsymbol{Q}}\to\bar{\boldsymbol{Q}}_p$. For a finite abelian group G, let $\hat{G}=\mathrm{Hom}\,(G, \bar{\boldsymbol{Q}}^\times)$. For a positive integer n, let ζ_n be a primitive n-th root of unity in $\bar{\boldsymbol{Q}}$.

§1. Put $m=5, 7$ or 11. Let p be an odd prime satisfying $(p, m\varphi(m))=1$, where φ is the Euler's φ-function. We use the following notations: $k=\boldsymbol{Q}\,(\cos\,(2\pi/m))$, $H=\mathrm{Gal}\,(k/\boldsymbol{Q})$, $K=k(\zeta_p)$, $G=\mathrm{Gal}\,(K/\boldsymbol{Q})$. Let ω be the Dirichlet character modulo p satisfying $\omega(a)\equiv a \bmod \mathfrak{p}$ for all integers a, $(a,p)=1$. For $\phi\in\hat{G}$, we identify ϕ with the primitive Dirichlet character attached to ϕ by class field theory. Then

$$\hat{G}=\{\psi\omega^i \mid \psi\in\hat{H},\ i \bmod (p-1)\}.$$

We say that $\phi\in\hat{G}$ is imaginary if ϕ (complex conjugation) $=-1$. Let $\hat{G}^-$ be the set of imaginary characters of G. For a positive integer i and for $\phi\in\hat{G}$, let $B_i(\phi)$ be the i-th generalized Bernoulli number associated with ϕ. For $\phi\in\hat{G}$, let Φ be the $\boldsymbol{Q}_p$-irreducible character of a representation of G which has ϕ as a $\bar{\boldsymbol{Q}}_p$-irreducible component. Then the orthogonal idempotent $e(\Phi)$ attached to Φ lies in the group ring $\boldsymbol{Z}_p[G]$ since $(p, [K:\boldsymbol{Q}])=1$. Let A be the p-Sylow subgroup of the ideal class group of K. We regard A as an additive group on which $\boldsymbol{Z}_p[G]$ acts naturally.

Our main result is the following

Theorem 1. *Let* $\phi\in\hat{G}^-$. *Then* $B_1(\phi^{-1})\equiv 0 \bmod \mathfrak{p}$ *if and only if* $e(\Phi)A\neq 0$. *In other words, let* $\psi\in\hat{H}$ *and let* i *be an even integer with* $2\leqq i\leqq p-1$. *Then* $B_i(\psi^{-1})\equiv 0 \bmod \mathfrak{p}$ *if and only if* $e(\Psi\omega^{1-i})A\neq 0$, *where*