38. *On Inclusion Relations between Two Methods of Summability*

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1. Introduction. Let $A=(a_{mn})$ be an infinite matrix. For a given sequence $\{s_n\}$, we set

$$\sigma_m=\sum_{n=0}^{\infty} a_{mn}s_n,$$

which is called the (A) mean of the sequence $\{s_n\}$. If the sequence $\{\sigma_m\}$ is convergent to s, then the sequence $\{s_n\}$ is said to be summable (A) to sum s. If any convergent sequence is necessarily summable (A), then the method of summability (A) is said to be *conservative.* If the sequence $\{\sigma_m\}$ is absolutely convergent, that is,

$$\sum_{m=0}^{\infty} |\sigma_m-\sigma_{m+1}|<+\infty,$$

then the sequence $\{s_n\}$ is said to be absolutely summable (A), or shortly summable $|A|$. If any absolutely convergent sequence is necessarily summable $|A|$, then the method of summability (A) is said to be *absolutely conservative.*

The purpose of this note is to solve the following problems.

(I) If the method of summability (A) is conservative, then is the method (A) absolutely conservative?

(II) If the method of summability (A) is absolutely conservative, then is the method (A) conservative?

In § 2, we show that these problems are negatively solved. In § 3, we prove some theorem concerning the problem (I). When (A) and (B) are methods of summability, we say that the method (B) includes the method (A) and use the notation $(A)\subseteqq(B)$, if any sequence summable (A) is necessarily summable (B). We shall now consider the following problems, in which the method (A) is not the unit matrix method, analogous to the above problems (I) and (II).

(I′) If $(A)\subseteqq(B)$, then is it true that $|A|\subseteqq|B|$?

(II′) If $|A|\subseteqq|B|$, then is it true that $(A)\subseteqq(B)$?

In § 4, we show that these problems are also negatively solved.

2. Concerning the problems (I) and (II), we state the following theorems.

Theorem 1. *There exists a method of summability (A) such that the method (A) is conservative but not absolutely conservative.*