35. *Deformation of Linear Ordinary Differential Equations. II*

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In the preceding note [1], we have developed the theory of isomonodromy deformation of linear ordinary differential equations. In particular we defined the τ function for each isomonodromy family, which is a generalization of the theta function in the theory of abelian functions.

In this note we deal with a transformation which changes the exponents of formal monodromy by integer differences (Schlesinger transformation). We also consider the ratio of the transformed τ function to the original one (τ quotient). Finally we shall give elementary examples of τ functions which corresponds to soliton and rational solutions in the theory of inverse scattering.

We use the same notations as [1].

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1. Given an $m\times m$ matrix $Y(x)$ with monodromy property in the sense of [1], we can construct another matrix $Y'(x)$ with the same monodromy data except for integer differences in the exponents of formal monodromy. Schlesinger [2] considered such a transformation in the case of regular singularities. His construction applies equally to the irregular singular case.

Choose integers l^{ν}_{α} ($\nu=1,\cdots,n,\infty$; $\alpha=1,\cdots,m$) satisfying the condition (the Fuchs' relation) $\sum_{\nu=1,\cdots,n,\infty}\sum_{\alpha=1}^{m} l^{\nu}_{\alpha}=0$, and set $L^{(\nu)}=(\delta_{\alpha\beta}l^{\nu}_{\beta})_{\alpha,\beta=1,\cdots,m}$. A transformation $Y'(x)=R(x)Y(x)$ from $Y(x)$ to $Y'(x)$ is called the Schlesinger transformation of type $\begin{Bmatrix}\infty & a_1 & \cdots a_n\\ L^{(\infty)} & L^{(1)} & \cdots L^{(n)}\end{Bmatrix}$, if it preserves the monodromy data except for the change of exponents of formal monodromy $T^{(\nu)}_0\mapsto T^{(\nu)}_0+L^{(\nu)}$.

The condition for $R(x)$ so that $Y'(x)$ is the desired matrix is the following.

(1) $$R(x)\hat{Y}^{(\infty)}(x)x^{L^{(\infty)}}=\hat{Y}^{(\infty)\prime}(x),$$

(2) $$R(x)G^{(\nu)}\hat{Y}^{(\nu)}(x)(x-a_\nu)^{-L^{(\nu)}}=G^{(\nu)\prime}\hat{Y}^{(\nu)\prime}(x)$$

with an invertible matrix $G^{(\nu)\prime}$ ($\nu\neq\infty$),