34. *Deformation of Linear Ordinary Differential Equations. I*

By Michio JIMBO and Tetsuji MIWA
Research Institute for Mathematical Sciences, Kyoto University

(Communicated by Kôsaku YOSIDA, M. J. A., April 12, 1980)

In this article we report on the general theory of isomonodromic deformations for a system of linear ordinary differential equations (1), having irregular singularities of arbitrary rank. A general scheme for such deformations was constructed by L. Schlesinger [1] for equations with regular singularities, and was recently extended by K. Ueno [2], B. Klares [10] to the case admitting irregular singularities. The main results of the present note are (i) proof of complete integrability of the nonlinear deformation equations (§§ 2–3), and (ii) introduction of the notion of τ function (§ 4).

Details of this and the forthcoming note II will be published elsewhere.

We wish to thank Prof. M. Sato, Drs. K. Okamoto, K. Ueno and also Profs. H. Flaschka, J. L. Verdier for many invaluable discussions.

1. Let $a_1, \cdots, a_n, a_\infty = \infty$ be distinct points on $\boldsymbol{P}^1$. We consider a system of linear ordinary differential equations with rational coefficients

$$(1)\qquad \frac{dY}{dx} = A(x)Y, \qquad A(x) = \sum_{\nu=1}^{n}\sum_{k=0}^{r_\nu} \frac{A_{\nu,-k}}{(x-a_\nu)^{k+1}} - \sum_{k=1}^{r_\infty} A_{\infty,-k} x^{k-1}$$

where $A_{\nu,-k}$ are $m \times m$ constant matrices. We set $A_{\infty 0} = -\sum_{\nu=1}^{n} A_{\nu 0}$. The leading coefficients $A_{\nu,-r_\nu}$ at $x = a_\nu$ are assumed to be diagonalized as

$$(2)\qquad A_{\nu,-r_\nu} = G^{(\nu)} T^{(\nu)}_{-r_\nu} G^{(\nu)-1} \qquad (\nu = 1, \cdots, n, \infty)$$

$T^{(\nu)}_{-r_\nu}$: diagonal with eigenvalues mutually distinct (if $r_\nu \geq 1$) or distinct modulo integers (if $r_\nu = 0$).

In the sequel we assume $A_{\nu,-r_\nu} = T^{(\nu)}_{-r_\nu}$ and choose $G^{(\infty)} = 1$. Along with (1) we consider equivalent systems with diagonalized leading term at $x = a_\nu$

$$(3)\qquad \frac{dY^{(\nu)}}{dx} = A^{(\nu)}(x) Y^{(\nu)}, \qquad A^{(\nu)}(x) = G^{(\nu)-1} A(x) G^{(\nu)}.$$

Equation (1) is specified by the following data

$$(4)\qquad a_\nu, A_{\nu 0}, \cdots, A_{\nu,-r_\nu+1}, T^{(\nu)}_{-r_\nu}, G^{(\nu)} \qquad (\nu = 1, \cdots, n);\ A_{\infty -1}, \cdots, A_{\infty - r_\infty}.$$

We denote by $\mathfrak{M}$ the affine manifold of "singularity data" (4).

Equation (3) has a unique formal solution of the following form ([5][6]):