30. *Cross Ratios as Moduli of Cubic Surfaces*

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1. It is evident from the Cartan's definition of the simple Lie group E_6 that the set of 27 lines upon the general cubic surface in P_3 can be put into a one to one correspondence with the set L of weights of the 27 dimensional representation of E_6 under which the triplets of intersecting lines correspond exactly to the triplets of weights whose sum is equal to 0. (There are two of the representations of this dimension which we regard as the same since they are transposed by the outer automorphisms of E_6.) Such a correspondence is called an *isomorphism* of both sets. By a *distinguished cubic surface* we mean pair (S, α) of a non-singular cubic surface S and an isomorphism α of L onto the set of lines upon S. Two distinguished surfaces (S, α), (S', α') are called *isomorphic* if there is a biregular morphism β of S to S' such that $\beta_* \circ \alpha = \alpha'$ where β_* is the bijection of the lines on S to the lines on S' induced by β. (The isomorphism β is unique, since it is determined by $\beta_* = \alpha' \circ \alpha^{-1}$.) The purpose of this note is to realize the set M of the isomorphism classes of distinguished cubic surfaces as an algebraic manifold and to obtain one of its natural completions by using the cross ratios that Cayley first considered for the cubic surface [1]. We shall now give the following remark since, throughout this note, the emphasis is put on the natural actions of $W(E_6)$ over various objects: The Weyl group $W(E_6)$, acting (transitively) on L, operates on the set of lines on S for every distinguished cubic surface (S, α) through the isomorphism α. Just in this sence $W(E_6)$ is classically called the automorphism group of the 27 lines upon the general cubic surface. $W(E_6)$ further operates on M if one requires $g \in W(E_6)$ to send the isomorphism class of (S, α) to that of $(S, \alpha g^{-1})$.

2. A projective plane that meets a non-singular cubic surface in the union of three lines is called a *tritangent* of the surface. Two tritangents are called *colinear* if their intersection lies entirely on the surface. Through one line on the surface there pass exactly five tritangents. Given four colinear tritangents of the surface, one can consider the cross ratios associated with them, since the totality of planes through one line in P_3 can naturally be regarded as the projective line. We regard all such cross ratios as the invariants of the cubic surface; more precisely, we regard them as k-valued functions