

23. Rational Maps to Varieties of Hyperbolic Type^{*)}

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§0. Introduction. In this paper we prove a finiteness theorem for the set of dominant rational maps to a variety of hyperbolic type. Let k be an algebraically closed field of characteristic zero. In this paper we assume all varieties are defined over k .

First we recall the definition of the Kodaira dimension. Let X be a smooth algebraic variety, then by Nagata and Hironaka there exists a complete smooth algebraic variety \bar{X} such that $D_X := \bar{X} - X$ is a divisor with normal crossings. Let K_X be the canonical divisor of \bar{X} and Φ_m the rational map of \bar{X} which is associated with the linear system $|m(K_X + D_X)|$.

Definition 1 ([3] and [4]). The logarithmic Kodaira dimension $\bar{\kappa}(X)$ of X is

$$\begin{cases} \sup_{m>0} \dim \Phi_m(\bar{X}), & \text{if } |m(K_X + D_X)| \neq (0) \text{ for some } m \in \mathbb{N}, \\ -\infty & , \text{if } |m(K_X + D_X)| = (0) \text{ for every } m \in \mathbb{N}. \end{cases}$$

If X is complete, $\bar{\kappa}(X)$ is denoted by $\kappa(X)$ and is called the Kodaira dimension of X . X is said to be of elliptic type, of parabolic type, and of hyperbolic type, if $\bar{\kappa}(X) = -\infty, 0$, and $\dim(X)$, respectively. Algebraic varieties of hyperbolic type are also called of general type.

This notion of hyperbolicity is different from that of [5]. But it is known that a smooth algebraic variety of hyperbolic type is measure-hyperbolic in the sense of [5] (cf. [8]). The Kodaira dimension is an important bi-rational invariant to classify algebraic varieties (cf. [10]).

Definition 2. Let X and Y be algebraic varieties. A rational map $f: X \rightarrow Y$ is said to be a strictly rational map, if there exists a proper bi-rational morphism $\pi: X' \rightarrow X$ such that $f \circ \pi$ is a morphism. f is said to be dominant, if $\dim(f \circ \pi)(X') = \dim(Y)$.

Our main theorem is as follows:

Theorem. *Let X be a smooth algebraic variety and Y a smooth algebraic variety of hyperbolic type. Then the set of dominant strictly rational maps of X to Y is finite.*

The following varieties are examples of varieties of hyperbolic type.

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