15. *Piecewise Linear Dehn's Lemma in 4-Dimensions*

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§ 1. **Statement of result.** Let W be a compact 4-manifold with non-empty boundary $\partial W = M$. Throughout this note, we shall adopt the convention that the handle decomposition of W possesses no 4-handles and only one 0-handle.

In [4], Norman has given a number of cases for which the analogue of Dehn's lemma in 4-dimensions works. See also Fenn [2]. Some examples for which such an analogue fails can be seen in [1] and [3]. Our version of Dehn's lemma is as follows:

Theorem. *Let W be a compact 4-manifold with non-empty boundary $\partial W = M$, and let $h: (D^2, S^1) \to (W, M)$ be a proper map whose restriction to S^1 is an embedding. Suppose that W admits a handle decomposition without 1-handles (hence is simply-connected), then h is homotopic to a PL-embedding keeping the boundary fixed. In particular, every loop on M bounds a PL-disc in W.*

§ 2. **Key lemma.** For a compact 4-manifold N with boundary $\partial N = M \cup M'$ (disjoint), the triad $(N; M, M')$ always admits a self-indexing Morse function $f: N \to [0, 4]$ such that $f^{-1}(0) = M$ and $f^{-1}(4) = M'$. Let D be a proper smooth submanifold of N of codimension two.

Lemma. *D can be isotopically deformed in N keeping the boundary fixed so that it satisfies the following*

(1) *$g = f \mid D: D \to [0, 4]$ is also a Morse function.*

(2) *For a critical point of g of index 1, its critical value is less than 2.*

Proof. The condition (1) is attained by small perturbation of D, because it is a smooth submanifold. Then, one may assume that there are no critical points of f on D, and no critical value of g is $1, 2$ or 3. Next, push down (or push up) a small neighborhood on D of each critical point of g of index 0 (or index 2) so that its critical value turns out to be less than 1 (or greater than 3). Then, it follows from the general position lemma that the same procedure as above makes the critical value of each critical point of g of index 1 less than 3.

Take a gradient like vector field of f on N, which defines the core of a 2-handle of N in $f^{-1}([a, 2])$ for some a $(1 < a < 2)$. This also defines the co-core of it in $f^{-1}([2, b])$ for some b $(2 < b < 3)$. Let C denote