14. *Non-Immersion and Non-Embedding Theorems for Complex Grassmann Manifolds**)

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Introduction. The purpose of the present paper is to prove non-immersion and non-embedding theorems for the complex Grassmann manifolds $G_{k,n-k}=U(n)/U(k)\times U(n-k)$ by making use of an index theorem due to Atiyah-Hirzebruch [1]. We denote by $X\subseteq R^q$ (or $X\subset R^q$) the existence of immersion (or embedding) of a differentiable manifold X into the Euclidean space R^q respectively. Let $\alpha(q)$ denote the number of 1's in the dyadic expansion of an integer q. Then our results are stated as follows:

Main Theorem. *Let* $2m=2k(n-k)$ *be the dimension of* $G_{k,n-k}$ *and let* $r=\sum_{j=1}^{k}(\alpha(n-j)-\alpha(j-1))$. *Then,*

(a) (i) $G_{k,n-k}\not\subset R^{4m-2r}$, (ii) $G_{k,n-k}\not\subseteq R^{4m-2r-1}$.

(b) *If n is odd, then $m=k(n-k)$ is even and*

(i) *if* $r\equiv 3 \pmod 4$ *then* $G_{k,n-k}\not\subset R^{4m-2r+2}$,

(ii) *if* $r\equiv 2$ *or* $3 \pmod 4$ *then* $G_{k,n-k}\not\subseteq R^{4m-2r+1}$,

if $r\equiv 1 \pmod 4$ *then* $G_{k,n-k}\not\subseteq R^{4m-2r}$.

These are generalizations of results for complex projective spaces investigated by Atiyah-Hirzebruch [1], Sanderson-Schwarzenberger [5] and Mayer [4] and of the results for some complex Grassmann manifolds obtained by Sugawara [6].

This paper is arranged as follows. In § 1, the index theorem for immersion and embedding due to Atiyah-Herzebruch [1] and Mayer [4] are recalled. § 2 is devoted to show the computability of some Todd genus for complex homogeneous spaces G/U. We prove Main Theorem in § 3 and exhibit Table I of r for some n and k.

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§ 1. Index theorems. Let X^{2m} be a closed connected oriented differentiable manifold of $2m$ dimension. Let $\{\hat{A}_j(p_1,p_2,\cdots,p_j)\}$ be the multiplicative sequence of polynomials [3, § 1] with $(z/2)/\sinh(z/2)$ as characteristic power series and let $\hat{\mathcal{A}}(X)$ be the cohomology class $\sum_{j=0}^{[m/2]}A_j(p_1(\xi),\cdots,p_j(\xi))$ of the tangent bundle $\xi=\tau(X)$ of X. For any $z\in H^*(X,Q)$ and $d\in H^2(X,Q)$, we define $\hat{A}(X,d,z)=\{ze^d\hat{\mathcal{A}}(X)\}[X]$. Let

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